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INTERNATIONAL COMPETITION FOR TECHNOLOGICAL EFFICIENCY: METHODOLOGY OF NONPARAMETRIC MODELLING

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Abstract. The article is devoted to the methodology of nonparametric modelling of the dynamics of countries' efficiency indicators considering global technological progress. The research is based on the DEA method with two inputs. The proposed models are constructed in the "labour-intensity – capital-intensity" coordinate system. In the course of their development, countries sequentially cross the world technology frontiers of different orders, from less efficient to more efficient.

The elementary strategies for countries in the factor intensity plane of production are as follows: 1) movement in the opposite direction, regardless of the positions of more efficient countries; 2) movement towards the more efficient country, approaching which requires either minimal changes in the previous development vector or minimal changes in the current proportion between production factors.

To reflect global trends, theoretical lines are constructed, which are international tracks of technological progress. Each such track is a convex hull of states of countries and, unlike the world technology frontier, has a positive slope. If global development factors prevail over internal ones, countries will be divided into groups, each following its own technological track. A less developed country will choose a more advanced country as a model within a particular group. A country located on the last (most efficient) segment of its international technological track will move towards a specific virtual state.

Depending on the nature of the global technological trend, international tracks of technological progress can lead to the convergence of production factor proportions of countries from different groups or to their divergence. In the first case, all international tracks will end at the point of highest efficiency $\Omega(0,0)$. In the case of divergence of technological proportions, tracks that bypass this point are possible, bringing countries closer to the infinite productivity of one production factor at a finite level of productivity of the other.

Keywords: data envelopment analysis, the efficiency of national economies, technological progress, world (global) technology frontier, factors of production, technological proportions.

1. INTRODUCTION

1.1. Problem Definition

Human civilization is entering a new technological era characterized by the emergence of an increasing number of new economic sectors and the radical transformation of existing ones. In many cases, the complexity of cutting-edge technologies necessitates cooperation among multiple countries. A prominent example of this is the Artemis program, led by NASA, with the goal of returning humans to the Moon. This program involves European countries (including Ukraine), Japan, Canada, Australia, and the UAE. However, the absence of China among the program participants raises concerns about it evolving into a new "moon race," reminiscent of the one that took place in the 1960s between the United States and the Soviet Union. Technological competition has long been underway in fields such as nuclear energy, computer technology, and genetic engineering.

It is evident that participation in such projects is only feasible for developed countries with highly efficient economies. Other countries risk becoming outsiders and must rely on importing cutting-edge technology and high-tech products from more advanced nations. The theoretical representation of these processes has led to the development of numerous models of technological progress designed for comparative analysis of national economies. The quintessence of these studies has been the concept of the World (or Global) Technology Frontier — a theoretical line formed by countries with the best combinations of efficiency indicators. This article complements this concept by analysing possible directions of movement for leaders and outsiders on the plane of technological efficiency indicators.

1.2. Analysis of Recent Research and Publications

1.2.1. Parametric Approach to Technological Progress Modelling

Traditionally, the foundational method of econometric modelling involves seeking correlations among specific economic indices. Ağan (2022) employs this approach in her work, examining changes in 72 countries based on the Technological Achievement Index (TAI). The results presented by the author show an increase in the number of countries that could become leaders in the TAI ranking. Amavilah & Rodriguez Andres (2023) dedicate their study to understanding the extent to which empirical rules of technological progress align with the evolution of developing countries.

In other versions of the parametric approach, more specialized methods are applied. For instance, Mohamed et al. (2022) utilize the Error Correction Model (ECM) and Granger causality test. Their findings indicate a direct dependence of a developing country's economic growth stability on indicators of technological innovation. Ault & Spicer (2022) employ a series of qualitative comparative analysis of fuzzy sets (fs/QCA) to determine the influence of formal institutional conditions on informal entrepreneurship.

A number of recent works are focused on the differentiation of countries based on their levels of development and efficiency. Sichera & Pizzuto (2019) introduced a toolkit for clustering economic groups into convergence clubs using the Phillips-Sul regression test. Wang (2020), based on World Bank data for 217 countries from 2000 to 2019, demonstrates that technological progress can reduce income disparities among countries. Analysing data from 183 countries, Škare & Riberio Soriano (2021) find that globalization positively impacts the transfer and implementation of digital technologies. Their study utilized data on globalization indices, digital technology adoption, global competitiveness, and aggregate factor productivity across 183 countries.

1.2.2. Comparative Analysis of National Economic Efficiency Using the DEA Method

The non-parametric method of Data Envelopment Analysis (DEA) has been gaining popularity recently. An overview of the history and current trends in the application of DEA methods is provided in the works of Narayanan et al. (2022) and Panwar et al. (2022). Since the late 20th century, this method has been used at the macro level to compare different countries' national economies.

In the study by Lafuente et al. (2022), a specialized version of the input-oriented DEA model with a single constant input, referred to as the "benefit of the doubt" (BOD) approach, is employed. The authors use the Global Entrepreneurship Index (GEI) proposed in their previous work. They present the results of their research in coordinate systems such as "BOD-GEI – GDP per capita" and "BOD-

GEI – venture capital investment (% of GDP)". Lafuente et al. (2020) analyze two types of entrepreneurship based on Kirzner and Schumpeter. They construct a Global Technology Frontier in the "Capital-to-labour ratio – GDP-to-labour ratio" coordinate system.

Krüger (2020) analyzed the dynamics of the global technological frontier using the Malmquist Index. Mastromarco & Simar (2021) found no significant influence of human capital on its shifts.

A study of 75 countries conducted by Carracedo & Puertas (2022) using the intertemporal DEA method revealed no direct relationship between innovation efficiency and expenditures on innovation.

It is worth noting that DEA models can be complemented by parametric methods. For example, Braun et al. (2021) demonstrated that the optimal strategy for firms and countries depends on their distance from the World Technology Frontier. The authors identified parametric dependencies between the innovation strategy index and other indicators characterizing national economies, such as total factor productivity, human capital, expenditure in R&D (% of GDP), patents, income, and physical capital (per capita).

Mitropoulos & Mitropoulos (2023, 2022) examined the efficiency of national entrepreneurship systems using a stochastic DEA model, meta-frontier methods, and convergence clubs. Based on data from 30 countries from 2013 to 2018, they concluded that countries oriented towards efficiency are technological leaders in entrepreneurship, while countries oriented towards innovation are followers.

1.3. Formulation of Objectives for the Article

The parametric approach discussed above continues the tradition initiated by the classical Solow model of economic growth. Production functions typically contain technological progress multipliers. In the case of the Cobb-Douglas function, this is a single indicator, total factor productivity (TFP), and in the case of the CES function, there are two, corresponding to the two production factors. These multipliers can be seen as functions of other macroeconomic indicators that consider institutional features of countries and their interaction in the global market. Each of these indicators can be transformed into more detailed arguments. The production function obtained in this manner becomes a tool for comparative analysis of national economies and the basis for forecasting their economic development.

The increasing complexity of such econometric models and the lack of necessary statistical data have led to the emergence of an alternative nonparametric approach. As indicated by the review of relevant contemporary literature, the concept of the World Technology Frontier is well-suited for the comparative analysis of national economic efficiency. It can be used to determine the reasons for certain countries lagging global technological leaders. The World Technology Frontier is not constant; it can shift in positive and negative directions in specific segments. These shifts can be analyzed and predicted using parametric methods.

However, supplementing DEA models with parametric methods reduces their inherent advantages, namely mathematical simplicity, and the requirement for a small number of statistical indicators. Considering this, the proposed research aims to maintain the critical advantages of the classical DEA method while constructing models that have the potential to forecast the movement of national economies on the plane of technological efficiency indicators. It is understood that purely nonparametric models can only determine the directions of countries' movements, not their specific indicators for future periods.

In the proposed research, the following questions are posed and addressed:

- What are the possible strategies for a country's movement on the plane of production factor productivity indicators?
- How can these movements be described by functions of labour and capital productivity indicators?
- What form should the DEA analogues of these functions take?

2. MATERIALS AND METHODS

2.1. DEA Model of the Hierarchical System of World Technology Frontiers

The foundation of this research is based on the DEA method with two inputs. Due to the specificity of this method, the authors extensively employ its geometric interpretation. In line with the objectives of our study, a known coordinate system of specific production factor quantities " $L/Y - K/Y$" was selected, where Y represents the national production volume of a country measured by a particular statistical indicator. In contrast, L and K represent the quantities of its production factors (labour and capital). The authors' methodology for determining these quantities based on official statistical data for a single year is outlined in the work by Zagoruiko & Petkova (2022).
L/Y and K/Y are inverse labour and capital productivity indicators, denoted as Y/L and Y/K. In the chosen coordinate system, the technological state of each country is represented as a point, and the World Technology Frontier (WTF) is represented as the left lower part of the convex envelope of these states.
According to the authors, the entire set of countries under investigation can be divided into groups based on their generalized efficiency. Countries with the best combinations of technological efficiency indicators will be positioned on the world frontier of the first order, which is the lower left part of the envelope encompassing all countries. Frontiers of higher orders (with worse combinations of specific production factor quantities) will represent sequential envelopes of the remaining countries. The constructed world frontiers in this manner have a negative slope and do not intersect. Thus, they can be considered as isoquants of the efficiency function of the world (or regional) economy.
In their previous work, the authors presented the geometric interpretation of the hierarchical system model of world technology frontiers (Zagoruiko & Petkova, 2021). According to this model, the coordinate axes of the " $L/Y - K/Y$ " system are interpreted as the world frontier of the zeroth order, a virtual unattainable limit that countries can only approach. The extreme boundary positions on each world frontier are occupied by leader countries, which are superior in one of the efficiency indicators in their group. In the case where one country is a leader in both indicators, its world frontier will be analogous to the Leontief production function and will consist of vertical and horizontal rays. Outsider countries will occupy the extreme positions on the final frontier.

2.2. Initial Idea and Research Stages

Traditionally, it is believed that less developed countries should aim for a virtual state formed by the intersection of a radial ray and the first (and only) world technology frontier. This research introduces an alternative approach.
The study is structured as follows:
First, the elementary strategies of a country seeking to improve its technological efficiency are examined. These strategies are based on the supposition that a country on a more distant world frontier orients itself toward a specific advanced country, considering the limitations imposed by its current position or the trends in its previous development. Countries on the first order's world frontier have no example to follow and will move directly toward the finite point of technological progress $\Omega(0,0)$.
Next, the impact of global trends on the direction of national technological progress is posed and theoretically resolved. Formally, these trends can be described analytically using power or exponential functions. According to the authors, these functions are suitable for interpreting as limit trajectories toward which absolute development tracks will tend. Discrete analogues of the proposed technological progress functions are international technological progress tracks (ITTs) constructed using the DEA method. These tracks are convex envelopes of states of countries. Unlike the world technology frontier, they have a positive slope.
If global development factors prevail over internal ones, countries will be divided into groups, each moving along its own technological track. Within a particular group, a less advanced country will

choose a more advanced country as a model, not necessarily from the "neighbourhood" of the world frontier. A country located on the last (most efficient) segment of its international technological track will move towards a specific virtual state.

Depending on the nature of the global technological trend, international technological progress tracks can lead to the convergence of factor proportions in countries from different groups or to their divergence. In the first case, all international tracks will end at the highest efficiency point, $\Omega(0,0)$. In the case of divergence in technological proportions, tracks that bypass this point, bringing countries closer to infinite productivity of one production factor at a finite level of productivity of another, become possible.

2.3. Terminology and Notation

For convenience, let us introduce the following concepts and notations:

$$k = K/Y \qquad l = L/Y \qquad (1), (2)$$

$$\bar{k} = k/l = K/L \qquad \dot{k} = \mathrm{d}k/\mathrm{d}l \qquad (3), (4)$$

$$\bar{l} = l/k = L/K \qquad \dot{l} = \mathrm{d}l/\mathrm{d}k \qquad (5), (6)$$

where k, l – specific quantities of labour and capital; $\bar{k}$, $\dot{k}$ – average and marginal capital-to-labour ratios; $\bar{l}$, $\dot{l}$– average and marginal labour-to-capital ratios.

We will refer to the function $k = k(l)$ as the capital intensity function of technological progress and the function $l = l(k)$ as the labour intensity function. The factor of production, the specific amount of which is an argument of such a function, will be considered independent. Note that the independence of a factor in a particular function does not inherently make it a determinant of changes in the dependent factor (although it may turn out to be).

Through normalization, the quantities in both functions become dimensionless:

$$k(l): \quad k/k^{bas} = f_k(l/l^{bas}) \qquad (7)$$

$$l(k): \quad l/l^{bas} = f_l(k/k^{bas}) \qquad (8)$$

where k^{bas}, l^{bas} – certain base quantities of specific capital and labour. The increments $\mathrm{d}k$ and $\mathrm{d}l$ will be called absolute factor savings, and the quantities $\mathrm{d}k/k^{bas}$ and $\mathrm{d}l/l^{bas}$ – normalized savings. The process in which the rate of capital savings exceeds the rate of labour savings can logically be characterized as capital-saving technological progress, the opposite process as labour-saving progress, and the process in which the rates of factor savings are equal as neutral progress:

$$\vec{\mathcal{K}}: \quad \mathrm{dln}k/\mathrm{dln}l > 1 \quad \Leftrightarrow \quad \mathrm{d}k/\mathrm{d}l > k/l \qquad (9)$$

$$\vec{\mathcal{L}}: \quad \mathrm{dln}l/\mathrm{dln}k > 1 \quad \Leftrightarrow \quad \mathrm{d}l/\mathrm{d}k > l/k \qquad (10)$$

$$\vec{\mathcal{N}}: \quad \mathrm{dln}l = \mathrm{dln}k \qquad (11)$$

3. THEORETICAL RESULTS AND DISCUSSION

3.1. National Strategies for Technological Progress

Countries can approach the zero technological frontier in various ways. In this process, both productivity indicators should improve. This requirement can logically be referred to as a condition for technological progress:

$$k_f^X > k_{f-1}^X > \ldots k_1^X > 0 \quad , \quad l_f^X > l_{f-1}^X > \ldots l_f^X > 0 \qquad (12), (13)$$

where f is the number of the country's world technology frontier, denoted as X.

For country Λ, which is a leader in one or both efficiency indicators, the technological progress condition corresponds to tracks located in the sector bounded by the following rays: the lower vertical ray $L/Y = l^{\Lambda}$ and the left horizontal ray $K/Y = k^{\Lambda}$. Since the virtual country Ω serves as the benchmark for country Λ, the shortest track to approach it is a ray of constant factor proportions. As for the rest of the countries, each of them can move, disregarding the experience of other countries or following the direction towards one of the more efficient "neighbours" on the coordinate plane.

For a specific country X, the most straightforward development strategy is to follow the same direction as in the previous period. In the case of discrete time, the marginal capital intensity of labour (capital-to-labour marginal ratio) and the marginal labour intensity of capital (labour-to-capital marginal ratio) will have the following form:

$$\dot{k}^X = \Delta k^X / \Delta l^X \qquad \dot{l}^X = \Delta l^X / \Delta k^X \tag{14), (15}$$

Depending on whether these increments are calculated for the next or previous period, we will obtain different marginal quantities:

$$\dot{k}^X_{fut} = (k^X_{t+1} - k^X_t)/(l^X_{t+1} - l^X_t) \qquad \dot{l}^X_{fut} = 1/\dot{k}^X_{fut} \tag{16), (17}$$

$$\dot{k}^X_{pas} = (k^X_t - k^X_{t-1})/(l^X_t - l^X_{t-1}) \qquad \dot{l}^X_{pas} = 1/\dot{k}^X_{pas} \tag{18), (19}$$

where fut, pas are the future and past marginal quantities. The consistent direction of development for country X will be expressed in the equality of future marginal quantities to past ones:

$$\dot{k}^X_{fut} = \dot{k}^X_{pas} \qquad \dot{l}^X_{fut} = \dot{l}^X_{pas} \tag{20), (21}$$

However, country X can change its development vector towards a more efficient country. In this case, it is advisable to orient towards country M, the approach to which will require the slightest change in the direction of past development:

$$\min_{M=1,..m} \left| \ln \left(\frac{k^X_t - k^M_t}{l^X_t - l^M_t} : \frac{k^X_{t-1} - k^X_t}{l^X_{t-1} - l^X_t} \right) \right| \tag{22}$$

$$k^X_t > k^M_t, \qquad l^X_t > l^M_t, \qquad k^X_{t-1} > k^X_t, \qquad l^X_{t-1} > l^X_t$$

where M – the numbers of countries that satisfy the technological progress condition of country X.
An alternative to the strategy of minimal change in the development direction is the strategy of minimal change in current technological proportions. According to this strategy, country X will orient itself toward country N, approaching which will require the most minor change in the capital-to-labour average ratio (or the labour-to-capital average ratio).

$$\min_{N=1,..n} \left| \ln \left(\frac{k^X_t}{l^X_t} : \frac{k^N_t}{l^N_t} \right) \right| \tag{23}$$

where N – numbers of countries that satisfy the condition of technological progress for country X.
The geometric interpretation of both technological development strategies is presented in Fig. 1.

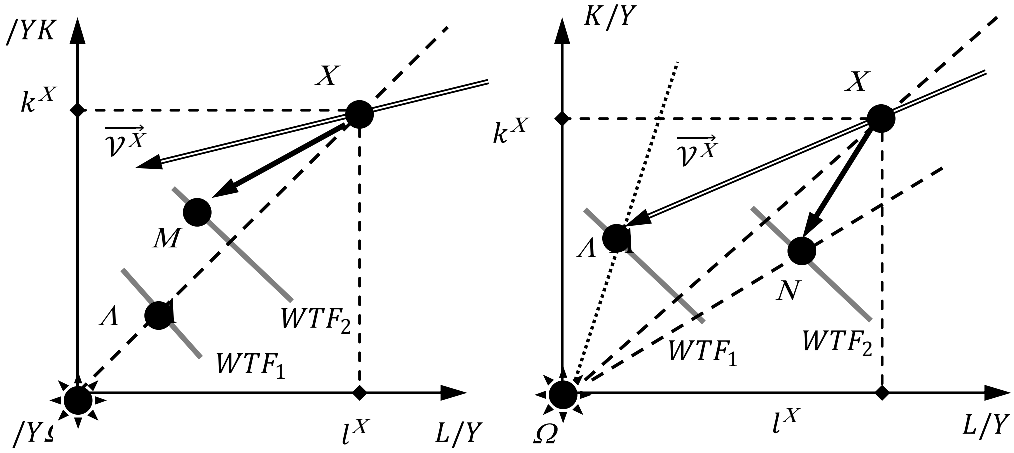

Figure 1. Comparison of country's technological development strategies.
$k = K/Y$, $l = L/Y$ – capital-to-GDP ratio, labor-to-GDP ratio; Ω – finite point of technological progress; ΩX, $\Omega\Lambda$, ΩN – rays of current technological proportions; $\overrightarrow{v^X}$ – vector of past development of country X; $WTF_{1,2}$ – world technology frontiers.

Source: The model is developed by I. O. Zagoruiko

On the left side of this figure, the strategy of minimal change in the direction of development is depicted. Country X changes its development vector and begins to move towards country *M*, even though country Λ is more efficient and has the same technological proportions as it.
The strategy of minimal change in technological proportions is illustrated on the right side of this figure. Country X changes its development vector and starts moving towards country *N*, even though country Λ is more efficient and located on its line of past development.

3.2. Global Technological Progress Trends

However, it is possible that the development of each individual country is significantly influenced by specific global trends, which can be described by corresponding mathematical functions in which the specific volume of one factor of production depends on the specific volume of another: $k = k(l)$) or $l = l(k)$. In this case, the coordinate plane will be "filled" with technological progress functions that have a similar form but differ in their parameters.
The discrete analogues of these functions will be *international technological tracks* (ITT). Country X will orient itself towards the country that is next on the same technological progress track. These tracks contain parts of sequential convex hulls of the set of countries, and their extreme boundary segments are determined by additional suppositions. The first international track contains a part of the convex hull of the entire set of countries. The second track contains a part of the hull of the remaining countries and so on.
All tracks of the same type are open polygonal chains with a positive slope and concave or convex relative to a certain *baseline technological proportion*. The baseline proportions are chosen as the ratio of production factor factors of the world leader (or leaders) in terms of productivity or the world outsider. If all countries are located on one side of the baseline, it transforms into a separate technological progress track. If the initial segments of international tracks are parallel to this baseline, technological progress will increasingly deviate countries from the baseline proportion between production factors. If the final segments of international tracks intersect one of the coordinate axes, the productivity of that production factor will reach a maximum with infinite productivity of the other.
The functions of technological progress can be represented as partial solutions to certain differential equations:

$$k(l): \qquad (\mathrm{d}k/k^{bas}):(\mathrm{d}l/l^{bas}) = f_k(l/l^{bas}) \tag{24}$$
$$l(k): \qquad (\mathrm{d}l/l^{bas}):(\mathrm{d}k/k^{bas}) = f_l(k/k^{bas}) \tag{25}$$

where k^{bas}, l^{bas} are certain base quantities of specific capital and labour volumes. The left-hand sides of these equations are *relative normalized factor savings*, and the right-hand sides are functions of the normalized specific volume of the independent factor.
The form of technological progress functions will be determined based on the following suppositions:
The first supposition is that, for any parameters of these functions, technological progress should culminate at the world zeroth-order boundary:

$$k(l): \qquad l = l^{min} \;\Rightarrow\; k = 0 \tag{26}$$
$$l(k): \qquad k = k^{min} \;\Rightarrow\; l = 0 \tag{27}$$

The second supposition is that a reduction in the specific volume of the independent factor of production increases the relative normalized savings of the dependent factor:

$$k(l): \qquad \frac{\mathrm{d}}{\mathrm{d}l}\left(\frac{\mathrm{d}k}{k^{bas}}:\frac{\mathrm{d}l}{l^{bas}}\right) < 0 \tag{28}$$
$$l(k): \qquad \frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{\mathrm{d}l}{l^{bas}}:\frac{\mathrm{d}k}{k^{bas}}\right) < 0 \tag{29}$$

The role of this supposition is that a reduction in the independent factor simplifies the savings of the dependent factor but complicates its own savings. The geometric interpretation of both suppositions is presented in Fig. 2.

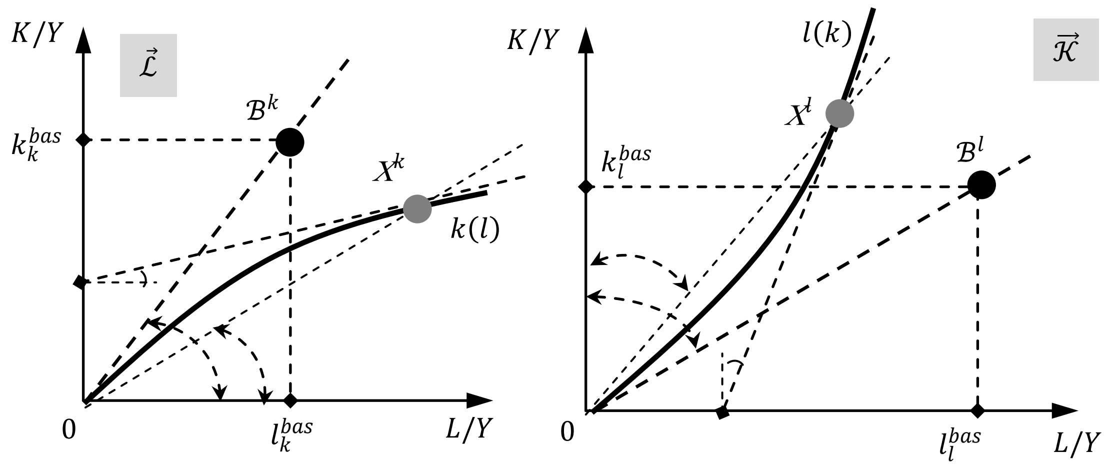

Figure 2. Suppositions about technological progress functions.
$k = K/Y$, $l = L/Y$ – capital and labour intensity of gross domestic product;
$k(l)$, $l(k)$– functions of capital intensity and labour intensity of technological progress;
$\mathcal{B}$ – points of basic intensity levels; $0\mathcal{B}^k$, $0\mathcal{B}^l$– rays of basic technological proportions;
$\vec{\mathcal{K}}$, $\vec{\mathcal{L}}$ – capital-saving and labour-saving technological progress.

Source: The model is developed by I. O. Zagoruiko

The left half of this figure depicts the function of the capital intensity of technological progress, while the right half shows the function of labour intensity. The marginal technological proportions are lower than the average ones, and the averages do not exceed the basic ones.

$$k(l):\qquad \mathrm{d}k/\mathrm{d}l < k/l < k_k^{bas}/l_k^{bas} \tag{30}$$

$$l(k):\qquad \mathrm{d}l/\mathrm{d}k < l/k < l_l^{bas}/k_l^{bas} \tag{31}$$

So, the function of capital intensity is entirely labour-saving, and the function of labour intensity is entirely capital-saving.

Let us consider elementary power and exponential functions that correspond to the two initial suppositions.

3.3. Unbounded power functions of technological progress

Power-type functions will correspond to a differential equation in which the ratio of normalized factor savings is inversely proportional to the normalized volume of the independent factor increased by one:

$$\frac{\mathrm{d}k}{k^{bas}} : \frac{\mathrm{d}l}{l^{bas}} = \frac{\kappa}{\left(\frac{l}{l^{bas}} + 1\right)^{1-\kappa}} \qquad \frac{\mathrm{d}l}{l^{bas}} : \frac{\mathrm{d}k}{k^{bas}} = \frac{\lambda}{\left(\frac{k}{k^{bas}} + 1\right)^{1-\lambda}} \qquad (32), (33)$$

where , λ are dimensionless positive parameters not exceeding one: $0 < \kappa \leq 1$, $0 < \lambda \leq 1$. These parameters determine the slope of the tangent line at the origin:

$$k(l):\quad l = 0 \quad \Rightarrow \quad \mathrm{d}k/\mathrm{d}l = \kappa \cdot k^{bas}/l^{bas} \tag{34}$$

$$l(k):\quad k = 0 \quad \Rightarrow \quad \mathrm{d}l/\mathrm{d}k = \lambda \cdot l^{bas}/k^{bas} \tag{35}$$

Partial solutions to this type of differential equation are unbounded power functions:

$$\frac{k}{k^{bas}} + 1 = \left(\frac{l}{l^{bas}} + 1\right)^{\kappa} \qquad \frac{l}{l^{bas}} + 1 = \left(\frac{k}{k^{bas}} + 1\right)^{\lambda} \qquad (36), (37)$$

As we can see, these equations are symmetric and can be transformed into each other. Thanks to this property, they can form a unified set with a common base state, which includes both capital-intensity functions $k(l)$, describing labour-saving technological progress, and labor-intensity functions $l(k)$, describing capital-saving progress. The geometric interpretation of this property is presented in Fig. 3.

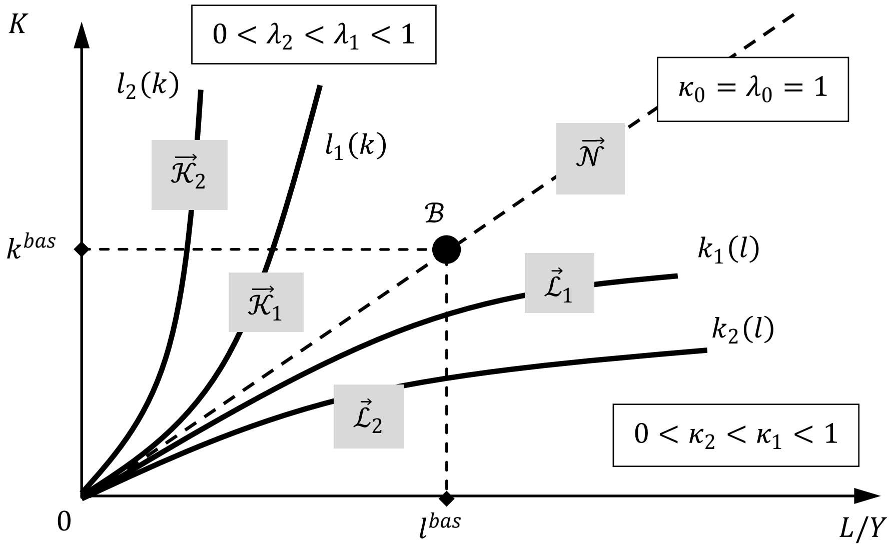

Figure 3. Unbounded power functions of technological progress.
$k = K/Y$, $l = L/Y$ – capital and labour intensity of gross domestic product;
$k(l)$, $l(k)$– functions of capital intensity and labour intensity of technological progress;
$\mathcal{B}$ – point of the basic technological state; $0\mathcal{B}$ – ray of the basic proportion between production factors; $\vec{\mathcal{N}}$, $\vec{\mathcal{K}}$, $\vec{\mathcal{L}}$, neutral, capital-saving, and labour-saving technological progress.

Source: The model is developed by I. O. Zagoruiko

In this figure, functions describing two different types of technological progress are depicted: above the ray of the basic technological proportion are labour-intensity functions describing capital-saving technological progress, and below this ray are capital-intensity functions describing labour-saving progress. The higher the exponent of the function, the closer it is to the ray of the basic technological proportion. A linear function of technological progress coincides with the ray of the basic proportion between production factors and represents neutral technological progress.

3.4. International Tracks of Technological Proportion Convergence

A discrete analogue of unbounded power functions is the international tracks of technological proportion convergence (ITT^{conv} – *convergence tracks*). The ray of the basic proportion passes through the point of the *ideal technological state* – $\mathcal{J}\left(l^{min}, k^{min}\right)$, coordinates of which represent a combination of the lowest existing levels of resource intensity. If this combination is virtual, the basic ray "splits" the set of countries into two parts. Countries located to the left (above) of the basic technological proportion ray follow capital-saving tracks – analogues of labour intensity functions $l(k)$. Countries located to the right (below) of the basic technological proportion ray follow labour-saving tracks – analogues of capital intensity functions $k(l)$. The initial tracks for both factors are closest to the basic proportion ray. During technological progress, countries' technological proportions converge toward the basic one.

Suppose one country is a leader in both productivity indicators in the world economy. In that case, its state is ideal and moves toward the point $\Omega(0,0)$, keeping the ratios of its production factors unchanged. In such a country, technological progress is neutral. In the case where the leader's technological proportion is extreme, only one type of factor-saving track remains in the world economy.

A graphical representation of international tracks of technological proportion convergence is presented in Fig. 4.

In the left part of this figure, the world technology frontier of the first order is formed by two countries, so the ideal technological state is virtual. The ray of the basic technological proportion passing through this point can be considered a virtual zero convergence track. In the case presented in the figure, the initial convergence tracks pass through the points of efficiency leaders and extreme technological proportions.

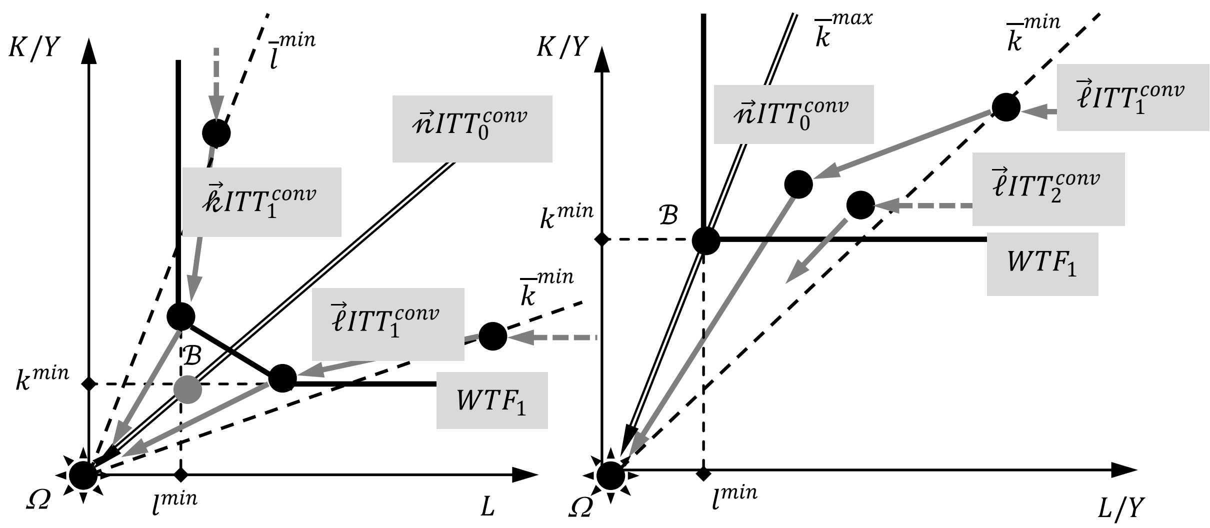

Figure 4. International Tracks of Technological Proportion Convergence.
$k = K/Y, l = L/Y$ – capital and labour intensity of gross domestic product;
$\vec{\ell}ITT^{conv}$, $\vec{k}ITT^{conv}$ – labour-saving and capital-saving tracks of technological proportion convergence; $\vec{n}ITT_0^{conv}$ – zero convergence track and neutral technological progress;
Ω – finite point of technological progress; $\mathcal{B}$ – point of the basic technological state;
$\Omega\mathcal{B}$ – ray of the basic proportion between production factors; WTF_1 – world technology frontier of the first order.

Source: The model is developed by I. O. Zagoruiko

In the right part of this figure, the world technology frontier of the first order is formed by one country, so its technological state is ideal. The country moves along a track of neutral technological progress. In this case, the world leader has the highest average capital-to-labour ratio, so all other countries move along labour-saving tracks. Additionally, the first labour-saving track passes through the point of the country with the lowest average capital-to-labour ratio.

In both cases, technological progress becomes neutral for countries located on the final segments of international tracks.

3.5. Exponential Functions with Slant Asymptotes

Alternative scenarios of global technological progress can be represented using indicator functions with slant asymptotes.

Like unrestricted power functions, these indicator functions can have a common base state and differ only in dimensionless parameters. Thanks to this property, they can also form a unified set, including both capital intensity functions $k(l)$and labour intensity functions $l(k)$.

Functions of the indicator type will correspond to a differential equation in which the ratio of normalized factor economies is equal to the inverse exponent of the normalized specific volume of the independent factor, increased by one:

$$k(l): \quad \frac{\mathrm{d}k}{k^{bas}} : \frac{\mathrm{d}l}{l^{bas}} = \kappa / \exp\left(\frac{l}{l^{bas}}\right) + 1 \tag{38}$$

$$l(k): \quad \frac{\mathrm{d}l}{l^{bas}} : \frac{\mathrm{d}k}{k^{bas}} = \lambda / \exp\left(\frac{k}{k^{bas}}\right) + 1 \tag{39}$$

Partial solutions to these differential equations consist of two types of indicator functions with slant asymptotes.

The following equations describe the functions of the first type:

$$k^I(l): \quad \frac{k}{k^{bas}} = \frac{l}{l^{bas}} + \kappa \cdot \left[1 - \exp\left(-\frac{l}{l^{bas}}\right)\right] \tag{40}$$

$$l^{I}(k): \qquad \frac{l}{l^{bas}} = \frac{k}{k^{bas}} + \lambda \cdot \left[1 - \exp\left(-\frac{k}{k^{bas}}\right)\right] \tag{41}$$

and functions of the second type are described by the following equations:

$$k^{II}(l): \qquad \frac{k}{k^{bas}} = \frac{l}{l^{bas}} - \kappa \cdot \exp\left(-\frac{l}{l^{bas}}\right) \tag{42}$$

$$l^{II}(k): \qquad \frac{l}{l^{bas}} = \frac{k}{k^{bas}} - \lambda \cdot \exp\left(-\frac{k}{k^{bas}}\right) \tag{43}$$

Where , λ are dimensionless positive parameters: $\kappa \geq 0$, $\lambda \geq 0$.

Both types of functions have slanted asymptotes. The slope angle of these asymptotes is equal to the slope angle of the basic technological proportion ray:

$$k, l \longrightarrow \infty \qquad \Longrightarrow \qquad \frac{\mathrm{d}k}{k^{bas}} = \frac{\mathrm{d}l}{l^{bas}} \tag{44}$$

The difference lies in the fact that each function of the first type has its own asymptote, shifted parallel to the basic ray by an amount proportional to the dimensionless parameter. This allows all of them to reach their origin. Moving along such a function, the economy increasingly deviates from the basic proportion between production factors.

On the other hand, all functions of the second type share a common asymptote - the ray of the basic technological proportion. Moving along such a function, the economy also deviates from the basic proportion between production factors. However, its technological progress ends on the axis of the independent factor.

The geometric interpretation of both types of power functions is presented in Fig. 5.

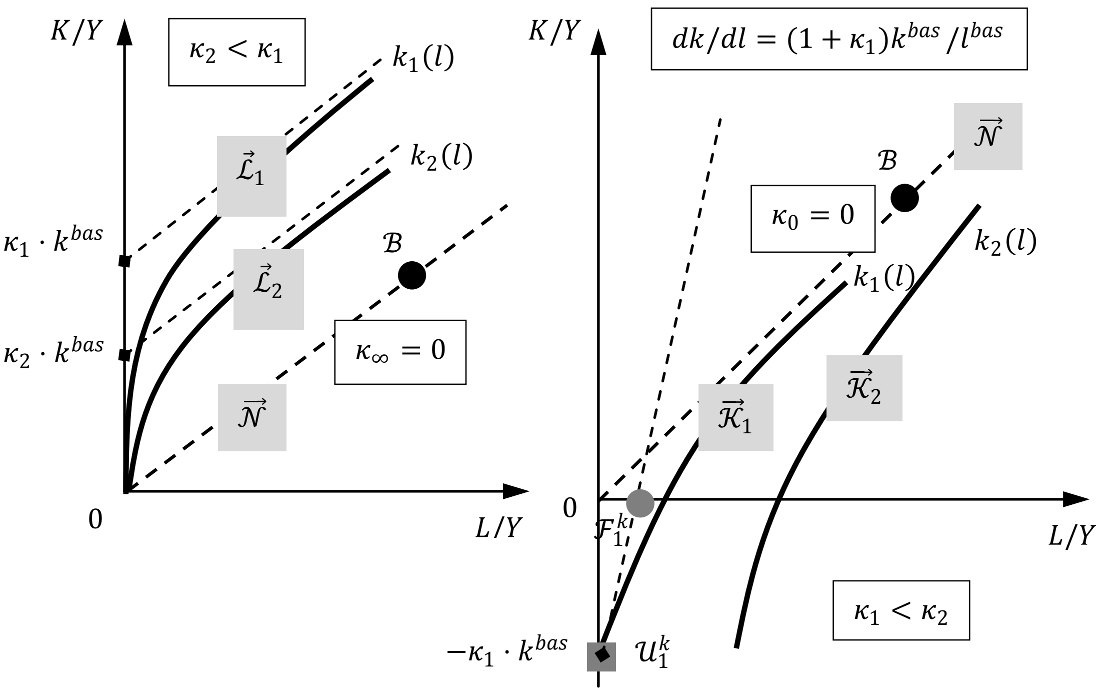

Figure 5. Power Functions of Technological Progress with Slant Asymptotes.
$k = K/Y$, $l = L/Y$ – capital and labour intensity of gross domestic product; $k(l)$ – functions of capital intensity of technological progress; $\mathcal{B}$ – the point of the basic technological state; $0\mathcal{B}$ – the ray of the basic proportionality between production factors; $\vec{\mathcal{N}}$, $\vec{\mathcal{K}}$, $\vec{\mathcal{L}}$ – neutral, capital-saving, and labor-saving technological progress; $\mathcal{U}_1^k\mathcal{F}_1^k$– the tangent ray to the function $k_1(l)$ at the point of its intersection with the capital intensity axis.

Source: The model is developed by I. O. Zagoruiko

In the left part of this figure, functions of the capital intensity of technological progress describing labour-saving technological progress are depicted. Their slanting asymptotes are parallel to the ray of the basic technological proportionality. Each function's asymptote intersects the dependent factor's axis at a level proportional to its basic value. The graphs of the functions themselves reach the origin.

In the right part of this figure, functions of capital intensity of technological progress are also shown. However, for these functions, the ray of the basic technological proportionality serves as a common asymptote, and they describe capital-saving technological progress. The graph of each function intersects the axis of the dependent factor at a negative level proportional to its basic value.

For a zero dimensionless parameter, both types of functions coincide with the ray of the basic proportionality between production factors.

3.6. International Tracks of Technological Proportion Divergence

The discrete analogue of power functions with slanting asymptotes of the first type is international tracks of technological proportion divergence leading to a state of infinite productivity of both production factors ITT^{inf} – *tracks to infinite productivity*). The second type's discrete analogue of power functions is international tracks of technological proportion divergence leading to a state of finite productivity of the independent production factor ITT^{fin} – *tracks to finite productivity*).

Since, in both cases, technological progress increasingly deviates countries from the ray of basic technological proportionality, the point of "horrible" state $\mathcal{H}(l^{max}, k^{max})$ is chosen as the basic state, with coordinates being a combination of the highest existing levels of resource intensity. Considering that the original continuous functions were located on one side of the basic ray, in the discrete model, it must be accurate and reflect an extreme technological proportion. This is possible only if the basic state is real, that is, the point of the world outsider – the country with the worst productivity indicators for both production factors. Therefore, a condition for applying both models of divergent tracks is the existence of a single-world outsider characterized by an extreme ratio of its production factors.

The initial sections of both types of divergent tracks are parallel to the ray of basic technological proportionality. However, in the case of ITT^{inf}, the first constructed track is the furthest from the basic ray, while in the case of ITT^{fin}, it is the closest to it.

If the tracks of ITT^{inf} are located to the left (above) of the basic ray, they will be discrete analogues of capital intensity functions $k(l)$ and reflect labour-saving progress. In contrast, they will be discrete analogues of labour intensity functions $l(k)$ and reflect capital-saving progress in the opposite case.

In the case of ITT^{fin}, the relationship between the original functions and the types of factor-saving progress will be opposite. If the tracks of ITT^{fin} run to the left (above) of the basic ray, they will be discrete analogues of labour intensity functions $l(k)$and reflect labour-saving progress. If, however, the tracks of ITT^{fin} run below (to the right) of the basic ray, they will be discrete analogs of capital intensity functions $k(l)$and reflect capital-saving progress.

The graphical representation of international tracks of technological proportion divergence is presented in Fig. 6.

Please note that the text contains specialized economic and mathematical terminology that may require additional context for a comprehensive understanding.

In both cases presented in this figure, one absolute leader and one absolute outsider in the world economy are heading towards the state of highest efficiency through neutral technological progress. (Although these progress tracks for the outsider are identical, for the sake of convenience, we will denote them according to the other tracks in the model). It should be noted that the track of neutral progress does not provide the outsider with a faster development in physical time compared to other countries.

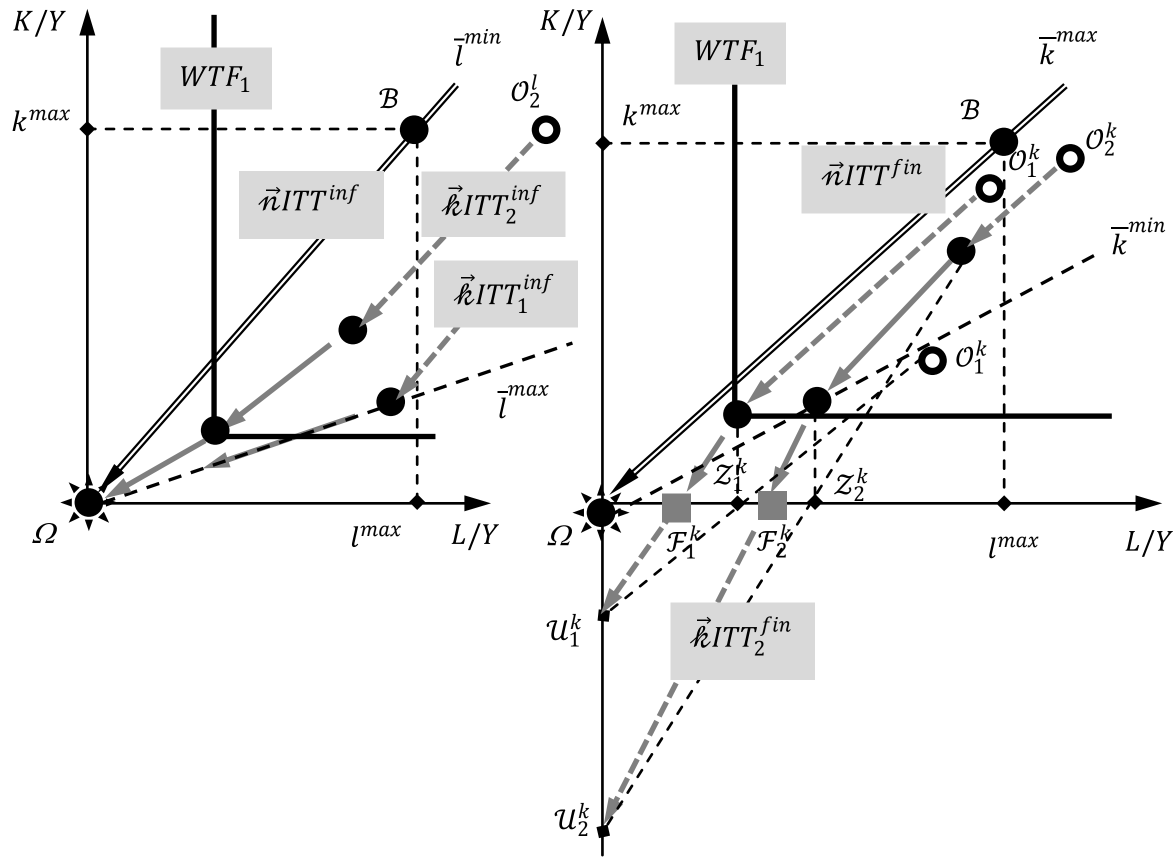

Figure 6. International Tracks of Technological Proportion Divergence.
$k = K/Y$, $l = L/Y$ – capital and labour intensity of gross domestic product;
$\vec{k}ITT^{inf}$, $\vec{k}ITT^{fin}$ – capital-saving tracks to infinite productivity of both production factors and finite labour productivity; $\vec{n}ITT$ – tracks of neutral technological progress; Ω – finite point of technological progress; $\mathcal{O}$ – infinitely distant virtual countries from which technological progress tracks begin; $\mathcal{U}$, $\mathcal{F}$ – ultimate and finite points of $\vec{k}ITT^{fin}$ tracks; $\mathcal{Z}$ – virtual state of the leader on the $\vec{k}ITT^{fin}$ track, corresponding to its current labour intensity and zero capital intensity; $\mathcal{B}$ – the basic country with the worst technological state; $\Omega\mathcal{B}$ – the ray of basic proportionality between production factors; WTF_1 – world technology frontiers of the first order.

Source: The model is developed by I. O. Zagoruiko

In the left part of the figure, capital-saving tracks $\vec{k}ITT^{inf}$ leading to the state of infinite productivity of both factors of production are depicted. The world outsider has the lowest level of average labour-capital ratio. The leader in terms of productivity indicators is located on the final segment of the second capital-saving track, which makes technological progress neutral for them. The same applies to the country with the maximum average labour-capital ratio. It is located on the final segment of the first capital-saving track, so technological progress also becomes neutral for them.

In the right part of the figure, capital-saving tracks $\vec{k}ITT^{fin}$ leading to a state of finite labour productivity are depicted. The world outsider has the highest level of average labour capital intensity. The leader in terms of productivity indicators is located on the final segment of the first capital-saving track. Thus, their final state has the lowest level of labour intensity. The country with the lowest level of labour capital intensity is on the final segment of the second capital-saving track. Thus, its final state has a higher labour intensity than the world leader.

Determining the directions of the final segments of tracks with finite labour productivity (in this case, for labour-saving tracks $\vec{k}ITT^{fin}$ and $\vec{\ell}ITT^{fin}$ is a separate issue that needs to be addressed in a general context.

Unlike the tracks ITT^{inf}, which converge to a single point, the tracks ITT^{fin} "bypass" the state of infinite productivity of production factors $\Omega(0,0)$. Their final segments initially intersect the axis of the independent factor and then virtually extend to the point of intersection with the axis of the dependent factor. The first (*finite*) point of intersection determines the highest level of independent factor productivity under conditions of infinite productivity of the dependent factor – $\mathcal{F}_j^k\left(l_j^{fin},0\right)$ or $\mathcal{F}_j^l\left(0,k_j^{fin}\right)$, respectively. The second (*ultimate*) point of intersection is in the negative region and defines the virtual level of dependent factor productivity under conditions of infinite productivity of the independent factor – $\mathcal{U}_j^k\left(0,k_j^{ult}\right)$ or $\mathcal{U}_j^l\left(l_j^{ult},0\right)$.

If the country $\Lambda_j\left(l_j^{min},k_j^{min}\right)$, which is a leader on its track, increases productivity only for the dependent factor, it will move towards a virtual state – $\mathcal{Z}_j^k\left(l_j^{min},0\right)$ or $\mathcal{Z}_j^l\left(0,k_j^{min}\right)$.

Based on these concepts, let's determine the direction of the final segment of the track of limited independent factor productivity. To do this, we will make the following suppositions.

First, let us assume that the last point $\mathcal{U}_j$ of the track ITT_j^{fin} must be such that the movement of each country X_j in its direction allows for the maximum increase in the productivity of the independent factor. This supposition is equivalent to minimizing the angle of this direction with respect to the axis of the independent factor:

$$k(l): \quad \left(k_j^X-k_j^{ult}\right)/l_j^X \to \min \qquad X=1,\dots\chi \tag{45}$$

$$l(k): \quad \left(l_j^X-l_j^{ult}\right)/k_j^X \to \min \qquad X=1,\dots\chi \tag{46}$$

where X is the country number on the track ITT_j^{fin}, starting from the leader ($X=1$); χ is the total number of countries on this track.

Secondly, let us assume that during the direct movement towards point $\mathcal{U}_j$, no outsider ($X\geq 2$) can surpass the level of independent factor productivity already achieved by the leader:

$$k(l): \quad l_j^X/\left(1-k_j^X/k_j^{ult}\right)\geq l_j^1 \qquad X=2,\dots\chi \tag{47}$$

$$l(k): \quad k_j^X/\left(1-l_j^X/l_j^{ult}\right)\geq k_j^1 \qquad X=2,\dots\chi \tag{48}$$

Together, these two suppositions ensure the convexity of the international technological track. On the other hand, such a DEA method for constructing the final segment of the limited productivity track is the only one that does not require more information than what is already available on the technological plane.

The made suppositions form a programming problem, the solution of which transforms the provided inequalities into equations and allows determining the intersection point of the international technological track with the axis of the dependent factor:

$$\mathcal{U}_j^k\left(0,k_j^{ult}\right): \quad -k_j^{ult}=k_j^2/\left(l_j^2/l_j^1-1\right) \quad l_j^2>l_j^1 \tag{49}$$

$$\mathcal{U}_j^l\left(l_j^{ult},0\right): \quad -l_j^{ult}=l_j^2/\left(k_j^2/k_j^1-1\right) \quad k_j^2>k_j^1 \tag{50}$$

$X=1$, the left-hand side of the above inequalities determines the lower bound of the reduction in the specific amount of the independent factor. Substituting the coordinates of the last point $\mathcal{U}_j$ into them will yield the desired quantity:

$$l_j^{fin}=l_j^1\Big/\left[1+\frac{k_j^1}{k_j^2}\cdot\left(\frac{l_j^2}{l_j^1}-1\right)\right] \tag{51}$$

$$k_j^{fin}=k_j^1\Big/\left[1+\frac{l_j^1}{l_j^2}\cdot\left(\frac{k_j^2}{k_j^1}-1\right)\right] \tag{52}$$

As we can see, the intersection point of the track ITT_j^{fin} with the axis of the independent factor is determined by the technological states of the leader Λ_j and the following country in line. The closer these countries are in terms of the independent factor's productivity, the lower the last point $\mathcal{U}_j$ is located in the negative region, and the greater the angle of inclination of the last segment of this track will be, thus the smaller the distance between the virtual points $\mathcal{F}_j$ and $\mathcal{Z}_j$. In other words, the similarity in the productivity of the independent factor between these countries can be considered an indication of their proximity to the limit of their productivity. In the same direction, an increase in the specific volume of the dependent factor operates in the second country. Its high value negatively affects the possibility of reducing the specific amount of the independent factor.

In the right part of Figure 6, the world leader in terms of productivity is the only real country on the first capital-saving track $\vec{k}\, ITT_1^{fin}$. Far beyond it is the virtual country $\mathcal{O}_1^k$. The segment on which it is located is parallel to the base technological proportion. Therefore, the ray $\mathcal{U}_1^k \mathcal{Z}_1^k$ can also be considered parallel to this ray. This determines the point $\mathcal{U}_1^k$, and the ray drawn from it to the state of the world leader is the point $\mathcal{F}_1^k$, thus the virtual minimum level of labour intensity on this track.

4. CONCLUSIONS AND RECOMMENDATIONS

In accordance with the objectives set in this study, the following results have been obtained:

1. Elementary strategies for a country's movement on the "labour-output – capital-output" plane "$L/Y - K/Y$" have been identified: 1) movement in the former direction, regardless of the positions of more efficient countries; 2) movement in the direction of a more efficient country, where approaching it requires either minimal changes in the previous development vector or minimal changes in the current proportion between production factors.
2. Three types of functions that can reflect global trends in changes in the productivity indicators of production factors have been proposed: 1) an unbounded power function that terminates at the origin – the point of infinite productivity of both production factors; 2) two exponential functions with oblique asymptotes, one of which "bypasses" the origin.
3. DEA analogues of these types of technological progress functions have been determined. The discrete analogue of a set of unbounded power functions differing in the exponent is the international tracks of convergence of technological proportions to the base level defined by the conditions of leader countries. The discrete analogue of a set of exponential functions that reach the origin is the international tracks of divergence of technological proportions leading to a state of infinite productivity of both production factors. The discrete analogue of a set of exponential functions that "bypass" the origin is the international tracks of technological proportions divergence leading to infinite productivity of only one production factor.

All proposed models require only the statistical data necessary to determine the specific volumes of two production factors. They can potentially forecast the movement of national economies on the "$L/Y - K/Y$" indicators plane. However, due to their purely non-parametric nature, the proposed models can only determine the directions of countries' movement, but not their indicators for specific future periods.

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Conflict of Interest statement

The authors declare no conflict of interest.

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МІЖНАРОДНЕ ЗМАГАННЯ ЗА ТЕХНОЛОГІЧНУ ЕФЕКТИВНІСТЬ: МЕТОДОЛОГІЯ НЕПАРАМЕТРИЧНОГО МОДЕЛЮВАННЯ

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Анотація. Статтю присвячено розробленню методології непараметричного моделювання динаміки показників ефективності країн із урахуванням світового технологічного прогресу. В основу дослідження покладено метод DEA з двома входами. Пропоновані моделі побудовані в системі координат «працемісткість – капіталомісткість». В процесі свого розвитку країни послідовно перетинають світові технологічні рубежі різних порядків – від менш ефективного до все більш ефективнішого. Елементарними стратегіями країни на площині інтенсивностей факторів виробництва є: 1) рух у колишньому напрямку, не зважаючи на положення більш ефективних країн; 2) рух у напрямку тієї більш ефективної країни, наближення до якої вимагає або мінімальної зміни попереднього вектору розвитку, або мінімальної зміни поточної пропорції між факторами виробництва. Для відображення глобальних тенденцій будуються теоретичні лінії – міжнародні шляхи технологічного прогресу. Кожен такий шлях є опуклою оболонкою станів країн, і, на відміну від світового технологічного рубежу, має додатний нахил. Якщо глобальні чинники розвитку переважатимуть над внутрішніми, країни поділяться на групи, кожна з яких буде рухатися власним технологічним шляхом. В межах окремої групи більш відстала країна обиратиме за зразок більш передову країну. Країна, що розташована на останній (найбільш ефективній) ділянці свого міжнародного технологічного шляху, рухатиметься в напрямку певного віртуального стану.
Залежно від характеру глобальної технологічної тенденції, міжнародні шляхи технологічного прогресу можуть вести до конвергенції пропорцій факторів виробництва країн з різних груп або до їх дивергенції. В першому випадку усі міжнародні шляхи завершуватимуться в точці найвищої ефективності . У випадку дивергенції технологічних пропорцій можливі шляхи, що оминають цю точку, наближаючи країни до нескінченної продуктивності одного фактору виробництва за скінченного рівня продуктивності другого.

Ключові слова: аналіз охоплення даних, ефективність національних економік, технічний прогрес, світовий (глобальний) технологічний рубіж, фактори виробництва, технологічні пропорції.