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Asymmetry indices of international position of countries: Geometric approach

Abstract. The article is devoted to the substantiation and testing of a new method for assessing the international position of countries. On the one hand, one of common methods of international comparative research is to construct a convex hull of the states of countries on the plane of certain indicators. Data Envelopment Analysis is the most well-known example of this approach. In particular, this method is used to build a world technology frontier. On the other hand, one of universal methods of initial indicators conversion is to normalise them. The method proposed in the article combines the construction of a convex hull on the plane of initial indicators with their min-max normalisation. The purpose of the study was to measure relative distances of countries to opposite sides of a certain hull of data. The problem is that at extremum points absolute distances to opposite sides of the original hull are equal to zero, and therefore relative distances cannot be determined. The authors solve this problem by constructing two secondary hulls of data, each of which allows determining of the asymmetry index by a certain coordinate. Opposite sides of the secondary hull are the midlines between the levels of opposite extrema and corresponding sides of the primary hull. A value that is reciprocal to the number of countries on the side of the primary hull, on which this extremum is located, is used as a weighting factor of the extremum. According to the proposed method, each country is characterised by a unique pair of asymmetry indices. This distinguishes it from the Data Envelopment Analysis method, according to which all countries on the boundary of efficiency are characterised by a unit distance. The proposed method has been tested on data for the countries of the European Union, Iceland and Switzerland for 2005, 2010, 2015 and 2020. The net international investment position (as a percentage of gross domestic product) and the difference between the stocks of immigrants and emigrants (as a percentage of the country's population excluding migrants) have been chosen as initial indicators. During the testing, the existence of a positive correlation between certain distances of countries on the plane of indices has been confirmed. It has been found that the global financial crisis of 2008 led to a radical shift in the hull of countries' states on this plane. Mapping of the international state of mean indices on the plane of initial indicators can be used in econometric models

Keywords: international comparative research; min-max normalisation; aggregate indices; hull of data; world technology frontier; international investment position; international migration

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Introduction

The comparative analysis of individual countries with others and groups of countries located in different parts of the world is one of important directions of international studies. The contradictory interaction of globalisation and competition in international relations, the impact of technological progress change relative indicators of individual countries and interstate integration associations. Thus, the rate of inflation and the level of unemployment in other countries have indirect effect on domestic economic policy. Other indicators – state of the balance of payments, net international investment position, flows and stocks of international migrants – determine both the foreign and domestic policies of states. International migration has turned into a global problem, and the level of institutional maturity of the market economic system and its political model are of great importance. Countries with authoritarian regimes abuse the conditions of global openness, use international economic relations as an instrument of foreign policy pressure.

The theory of indices, in particular composite ones, is one of the foundations of comparative analysis in economics. In modern economic literature, various aspects of constructing composite indices – normalisation of initial indicators, their weighting and aggregation – are analysed in detail. Transformed data are further used to construct various composite indices and in multi-criteria analysis. Thus, Ş.U. Arsu & T. Arsu (2023) have applied multi-criteria decision-making methods in the study of corporate sustainability of manufacturing companies. In this study, the authors use such popular normalisation methods as rescaling (min-max normalisation) and standardisation (Z-score normalisation). In the work of I. Stojanović *et al*. (2022), the elements of the initial decision matrix are normalised using the arithmetic mean of four traditional types of normalisation. V. Stojkoski *et al*. (2023) have used the Economic Complexity Index (ECI) method to investigate inclusive green growth in 98 countries. To normalise the matrix of revealed comparative advantages, the product of each of its original elements and the sum of all elements is divided by the product of the sums of elements by rows and columns. T. Jellema *et al*. (2020) have proposed synthetic indicators to assess the quality of macroeconomic statistics. These indicators are considered on the example of mirror data of counterparty countries regarding their mutual assets and liabilities. In the proposed bilateral asymmetry index, the difference of these mirror-opposite values is divided by their sum. R. Stellian & J.P. Danna-Buitrago (2022) have investigated the problem of choosing the form of the index of revealed comparative advantages (RCA). The authors have considered RCA indices of Balassa, Vollrath, Leromain-Orefice, as well as RCA indices based on hypothetical trade balances. To compare these indices, they propose a standardised method for assessing the quality of empirical measurements.

One of traditional visualisation methods is to use histograms. T.C.D. Echeverría *et al*. (2022) have used this method for a comparative analysis of economic indicators of the G20 countries for 2020. According to another method, the state of countries is represented as a point in the Cartesian coordinate system. Thus, S. Voitko & I. Grinko (2017) have conducted a comparative analysis of the sustainable development potential of Ukraine and countries in the peer-group. The authors visualise the results of their research on the planes of "GDP (gross domestic product) per capita – Quality of Life Index", "GDP per capita – Security of Life Index", "Index of Sustainable Development – Index of Innovation". Conventional radial systems, in which sectors of certain indicators (usually composite ones) are located along the circle, and the lengths of the radii map the values of these indicators, are an alternative method of visualisation. Thus, C. Melara-Gálvez & E. Morales-Fernández (2022) have used such radial diagrams for a comparative analysis of the competitiveness of Central American countries. The popular Doughnut Economy model is a variant of conditional radial systems. In it, the outer boundary of the ring is the boundary of economic development that does not harm the environment, and the inner boundary is the boundary determined by social needs. L.J. Kaivo-oja *et al*. (2022) have applied this method to a comparative analysis of ASEAN (Association of Southeast Asian Nations) countries. The application of graph theory becomes a new direction in the analysis of the world economy dynamics. Thus, Y. Abbas & A. Daouia (2023) have applied it in a study of the impact of news articles.

Modern scientific literature presents various works that use both classical and new econometric methods. I. Atanasova & T. Tsvetkov (2021) have constructed a regression model in which GDP per capita, the Gini coefficient, and the KOF Globalisation Index are linear functions of each other. S. Li & B. Wang (2020) have presented economic growth of the G20 countries as a quadratic function of social justice. E. Spyromitros & M. Panagiotidis (2022) have built a multifactorial model of the impact of corruption on economic growth in developing countries. Fuzzy set qualitative analysis is a new direction of international comparative research. Thus, H. Ding (2022) has applied this approach in the study of the dependence of national innovations on various economic, social and political factors.

Considering the above literature review, it is important to expand the set of indicators used to assess the international position of countries. According to the authors, existing (mainly algebraic) methods should be supplemented with methods characterising the geometry of the mutual position of countries on the plane of initial indicators and the plane of certain indices. Based on these considerations, the next goal of the study – to find and test an elementary geometric method of assessing the asymmetry of international position of both an individual country and the entire set of countries under study – has been set.

Materials and Methods

The min-max normalisation of the country's indicators relative to actual extrema of the studied set is the simplest assessment of the asymmetry of its international position. The disadvantage of these asymmetry indices is that each of them takes into account the state of only two countries and only by one coordinate. As a result, a change in one of the coordinates of "extreme" countries will not affect the indices of the rest of the countries by another coordinate. Similarly, the asymmetry index of a certain country by one coordinate will not change if another coordinate of its state changes.

It is possible to get rid of these and other similar disadvantages when building a convex hull of the states of countries – Conv$\mathbb{X}(z_1; z_2)$, where $\mathbb{X} = \{X_1, ..X_k, ...X_K\}$ – a set of countries; k – a country number; K – the total number of countries under study; z_1, z_2 – indicators of their state, which can acquire negative values. The coordinates of the points of this hull are theoretical extrema, which show in what range one of the country's indicators can change, provided the value of the second is constant. Now, with a similar shift in "extreme" countries, Conv$\mathbb{X}$ will change, which will also change both asymmetry indices of countries under study.

However, in such a model, elementary min-max normalisation leads to an indeterminate form of 0/0. Namely, extreme right and extreme left countries will be located simultaneously on upper and lower parts of the hull, and extreme upper and extreme lower ones – simultaneously on right and lower parts. This indeterminate form can be eliminated using the arithmetic mean of actual and theoretical extrema:

$$h_{1k}^{max} = \lambda_1^{max} \cdot g_1^{max} + (1 - \lambda_1^{max}) \cdot c_{1k}^{max} \; 0 < \lambda_1^{max} < 1; \quad (1)$$

$$h_{1k}^{min} = \lambda_1^{min} \cdot g_1^{min} + (1 - \lambda_1^{min}) \cdot c_{1k}^{min} \; 0 < \lambda_1^{min} < 1, \quad (2)$$

where c_{1k}^{min}, c_{1k}^{max} – coordinates of the endpoints of the horizontal chord connecting opposite sides of the Conv$\mathbb{X}$ hull; g_1^{min}, g_1^{max} – global (actual) extrema; $\lambda_1^{max}, \lambda_1^{min}$ – dimensionless parameters; h_{1k}^{min}, h_{1k}^{max} – coordinates of the ends of the horizontal chord connecting opposite lines of middle extrema.

The $\lambda_1^{max}, \lambda_1^{min}$ parameters should satisfy two conditions. First, if all countries are located on the same horizontal chord, then their hull will consist of "extreme" countries. In this case, both parameters should be equal to one. Second, if the number of countries on a certain side of the hull is infinite, then the arithmetic mean line will turn into a smooth curve that should coincide with the Conv$\mathbb{X}$ hull. In this case, the corresponding parameter should be equal to zero. These conditions are satisfied by the equalities:

$$\lambda_1^{min} \stackrel{def}{=} 1 / K_1^{min}; \quad (3)$$

$$\lambda_1^{max} \stackrel{def}{=} 1 / K_1^{max}, \quad (4)$$

where K_1^{min}, K_1^{max} – the number of countries on left and right sides of the hull, respectively.

Arithmetic mean lines constructed in this way will be located outside the original (primary) Conv$\mathbb{X}$ hull. Given this property, they can be called "horizons" of the studied set of countries. Visualisation of these horizons is presented in Figure 1.

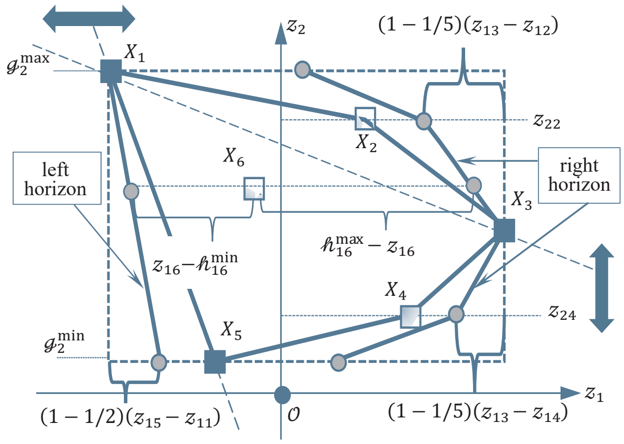

Figure 1. Left and right horizons of the convex hull of the states of countries

Notes: h_{16}^{min}, h_{16}^{max} – coordinates of the ends of the horizontal chord of the X_6 country; z_{16} – value of the z_1 indicator of the X_6 country; g_2^{max}, g_2^{min} – global extrema of the z_2 indicator

Source: authors' model

Left and right horizons together with the highest and lowest chords ($z_{2h}^{max} = \max\{z_{2k}\}$, $z_{2h}^{min} = \min\{z_{2k}\}$) form the secondary SConv($\mathbb{X}$, $h_1(z_2)$) hull, in which asymmetry indices will be determined by the first coordinate. Similarly, upper and lower horizons together with extreme lateral chords ($z_{1h}^{max} = \max\{z_{1k}\}$, $z_{1h}^{min} = \min\{z_{1k}\}$) form the secondary SConv($\mathbb{X}$, $h_2(z_1)$) hull, in which asymmetry indices will be determined by the second coordinate.

In the proposed study, the index of the country's distance to the right horizon of maxima has been determined as:

$$i_{1k} \stackrel{def}{=} \frac{h_{1k}^{max} - z_{1k}}{h_{1k}^{max} - h_{1k}^{min}} \Rightarrow 0 \leq i_{1k} \leq 1. \quad (5)$$

Hence, the index of the distance to the left horizon of minima is equal to:

$$1 - i_{1k} = \frac{z_{1k} - h_{1k}^{min}}{h_{1k}^{max} - h_{1k}^{min}}. \quad (6)$$

The studied set of countries can be characterised in different ways. According to the first method, the point of mean coordinates is first determined:

$$\overline{z_1} = \frac{1}{K}\sum_{k=1}^{K} z_{1k}; \quad (7)$$

$$\overline{z_2} = \frac{1}{K}\sum_{k=1}^{K} z_{2k}. \quad (8)$$

In the future, the $G(\overline{z}_1;\overline{z}_2)$ point constructed in this way will be called the general state of the studied set of countries. According to the second method, $\overline{i}_1$, $\overline{i}_2$ mean indices are first calculated:

$$\overline{i_1} = \frac{1}{K}\sum_{k=1}^{K} i_{1k}; \quad (9)$$

$$\overline{i_2} = \frac{1}{K}\sum_{k=1}^{K} i_{2k}. \quad (10)$$

The $R(\overline{\imath}_1; \overline{\imath}_2)$ constructed in this way characterises the relative state of the studied set of countries. The $G(\overline{z}_1; \overline{z}_2)$ point of the general state can be mapped onto the $\imath_1 O \imath_2$ plane of indices, and the $R(\overline{\imath}_1; \overline{\imath}_2)$ point of the relative state – onto the $z_1 O z_2$ plane of initial indicators.

Similarly, two more states can be mapped from one plane to another – the zero Z state (which is the origin of the coordinates on the plane of initial indicators) and the symmetrical S state (the state of equal indices).

Another way of general characterisation of the set of countries is that international indices are calculated not for the entire set of countries, but for a certain group occupying a "central" position. For this purpose, hulls are successively constructed for sets of countries, from which the countries located on previous hulls are excluded:

$$\begin{cases} \mathbb{X}_{-1} = \mathbb{X} \setminus \mathrm{Conv}\mathbb{X} \\ \mathbb{X}_{-2} = \mathbb{X}_{-1} \setminus \mathrm{Conv}\mathbb{X}_{-1} \\ \dots \\ \mathbb{C} \stackrel{\text{def}}{=} \mathbb{X}_{-s} = \mathbb{X}_{-s+1} \setminus \mathrm{Conv}\mathbb{X}_{-s+1} \neq \emptyset \\ \mathbb{X}_{-s-1} \stackrel{\text{def}}{=} \mathbb{X}_{-s} \setminus \mathrm{Conv}\mathbb{X}_{-s} = \emptyset \end{cases}. \quad (11)$$

The last non-empty $\mathbb{C}$ set in this series can be considered as the "central" hull of the studied set of countries. The mean state of the central $C(\overline{z}_1(\mathbb{C}); \overline{z}_2(\mathbb{C}))$ group of countries is considered the central state of the entire set. For the central C state, $\imath_{1C}$ and $\imath_{2C}$ distance indices can be calculated.

Additional information about the state of individual countries and their entire set can be obtained using the $\imath_1 O \imath_2$ space of indices. On this plane, all possible states of countries are located in a unit square. The point of intersection of its diagonals $S(0.5; 0.5)$ will be an international symmetrical state. Instead, the point of the zero Z state will no longer be constant and will change its coordinates with the change in the positions of the countries. The relative international $R(\overline{\imath}_1; \overline{\imath}_2)$ state will be the midpoint of national normalised states. The general international $G(\overline{z}_1; \overline{z}_2)$ state will be the normalised state of mean initial indicators. The point of the focused $F(\overline{\imath}_1(\mathbb{F}); \overline{\imath}_2(\mathbb{F}))$ state, where $\mathbb{F}$ – the last non-empty set remaining after successive subtraction of hulls will be an analogue of the point of the central state. The possibility of calculating Euclidean distances between countries and their distances to international states is the advantage of the plane of indices.

For the set of $\mathbb{X}(\imath_1; \imath_2)$ points, it is possible to construct a new convex hull – $\mathrm{Conv}\mathbb{X}(\imath_1; \imath_2)$. This hull may include countries that do not belong to the original $\mathrm{Conv}\mathbb{X}(z_1; z_2)$ hull. The area of the polygon formed by the $\mathrm{Conv}\mathbb{X}(\imath_1; \imath_2)$ hull can be used as an indicator of its shape. The area of the figure bounded by neighboring sides of the unit square and the $\mathrm{Conv}\mathbb{X}(\imath_1; \imath_2)$ hull characterises its curvature in the direction of the corresponding vertex.

The intersection of regions bounded by hulls of different periods can be interpreted as a "stability zone":

$$\mathfrak{X}_t \colon \partial \mathfrak{X}_t \stackrel{\text{def}}{=} \mathrm{Conv}\mathbb{X}(\imath_{1t}; \imath_{2t}); \quad (12)$$

$$\mathfrak{S}_T = \textstyle\bigcap_{t=1}^{T} \mathfrak{X}_t, \quad (13)$$

where $\mathfrak{X}_t$ – a continuous set, the boundary of which is identical to the primary hull of countries' states on the plane of indices.

The ratio of the area of the "stability zone" to the area of the corresponding region of a certain period will characterise its "inertia":

$$\jmath(\mathfrak{X}_t) = A(\mathfrak{S}_T)/A(\mathfrak{X}_t). \quad (14)$$

The method of secondary hulls and international states can be used in the analysis of various sets of countries and any of their indicators. In the proposed study, the countries of the European Union and Iceland with Switzerland, which are closely integrated with it, have been chosen as the object of research. Two initial indicators – net international investment position and net international migrant stock (the difference between the total number of immigrants and emigrants) – have been chosen as the subject of research. A positive value of the net international migrant stock means that the country's population has increased due to foreigners, and a negative value means that the country is losing population due to the emigration of its citizens. Both of these indicators are expressed in percentages. Traditionally, the net international investment position has been calculated as a percentage of the country's annual gross domestic product. The net international migrant stock has been expressed as a percentage of the country's population excluding migrants.

Selected indicators characterise the country's relations with the world in various ways. From a macroeconomic point of view, the net investment position is the result of international capital movement and the net migrant stock is the result of international human capital movement. The study has used statistical data from the United Nations (2020; 2022). Years of 2005, 2010, 2015 and 2020 have been chosen as the observed periods. According to the UNDESA (United Nations Department of Economic and Social Affairs) methodology, the migrant stock is calculated for the middle of the year.

Net investment position statistics have been obtained from Eurostat (n.d.a) and OECD Data Explorer (n.d.). Due to the lack of data, Norway and Liechtenstein are excluded from the studied set. For the comparability of time points, data on the net investment position at the end of the second quarter of the corresponding year are used. GDP statistics have been obtained from Eurostat (n.d.b) and Undata (n.d.) websites. At the time of data collection (April 2024), GDP figures for Bulgaria and Iceland for 2020 have been missing, so the sum of quarterly GDPs has been used instead.

Results

Based on initial statistical data, the coordinates of the countries are determined on the plane of initial indicators – "relative value of the net international investment position (z_1) – relative value of the net international migrant stock (z_2)" (Table 1). For each year, primary ($\mathrm{Conv}\mathbb{X}$) hulls of the states of countries on this plane are constructed. The number of countries on a certain side of the hull determines the value of the λ weighting factor, which is multiplied by the corresponding coordinate of the "extreme" country.

Table 1. Coordinates of countries on the plane of initial indicators

	2005		2010		2015		2020	
	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
Austria	−10.51	8.93	−3.37	10.69	4.47	13.52	16.44	15.86
Belgium	34.93	8.75	57.07	10.88	45.30	13.39	38.65	14.94
Bulgaria	−30.93	−10.92	−95.16	−13.71	−63.76	−14.69	−26.49	−22.06
Croatia	−54.94	−6.75	−92.58	−6.51	−78.25	−6.64	−50.85	−14.33
Cyprus	−73.99	−3.40	−122.28	2.66	−151.08	1.00	−123.00	1.64
Czechia	−21.59	−4.31	−42.94	−3.78	−30.14	−3.99	−16.34	−4.86
Denmark	−0.37	4.32	8.18	5.29	37.98	7.07	70.28	9.02
Estonia	−86.39	7.46	−75.71	5.77	−42.59	0.50	−21.77	−0.65
Finland	−15.54	−2.14	12.97	−1.23	6.46	0.63	−3.21	1.44
France	−3.14	9.32	−13.23	9.71	−14.93	10.30	−28.09	11.05
Germany	9.26	8.30	22.06	8.55	32.71	9.17	60.46	17.62
Greece	−69.48	3.26	−98.52	5.47	−134.87	4.42	−170.60	2.75
Hungary	−88.49	−1.03	−108.88	−0.78	−77.04	−1.34	−49.49	−1.42
Iceland	−72.57	−1.17	−627.92	0.93	−349.92	0.71	28.35	7.36
Ireland	−39.46	−5.12	−109.24	0.03	−187.47	0.58	−181.83	3.36
Italy	−17.35	2.32	−19.57	5.98	−17.32	5.77	−1.78	5.89
Latvia	−46.11	5.70	−82.94	1.99	−60.92	−6.01	−39.08	−8.48
Lithuania	−40.07	−6.66	−61.40	−11.99	−46.79	−14.86	−20.60	−19.17
Luxembourg	18.50	31.13	−7.11	29.39	74.14	56.01	75.51	65.09
Malta	35.07	−19.63	11.77	−17.18	34.57	−10.13	126.82	2.99
Netherlands	−4.15	6.51	1.32	6.62	45.22	7.51	103.16	9.21
Poland	−39.31	−5.71	−56.38	−8.10	−64.39	−9.13	−44.38	−10.66
Portugal	−66.52	−9.97	−107.51	−11.85	−118.21	−11.65	−105.14	−11.61
Romania	−24.14	−9.36	−63.14	−15.81	−52.29	−16.00	−42.93	−17.52
Slovakia	−38.36	−2.69	−59.03	−2.18	−59.90	−3.24	−66.26	−4.23
Slovenia	−7.04	5.47	−41.50	7.25	−34.00	5.48	−17.57	6.40
Spain	−59.62	7.56	−90.58	12.89	−89.66	11.40	−80.40	13.21
Sweden	−21.72	10.82	−14.98	12.99	2.06	15.77	12.96	20.04
Switzerland	113.82	23.18	118.50	25.24	60.13	26.78	104.74	28.92
Un. Kingdom	−4.78	3.27	−13.42	4.83	−9.64	7.09	−3.31	8.02

Notes: z_1 – the net international investment position at the end of the second quarter as a percentage of the country's GDP; z_2 – the net international immigrant stock as a percentage of the country's population excluding migrants. The states forming the primary hull on the plane of initial indicators are highlighted in gray

Source: prepared by the authors based on data from statistical sites

So, in 2005, Hungary, Switzerland, Malta and Luxembourg were the "extreme" countries. In 2010, Iceland, Switzerland, Malta and Luxembourg were the extrema of the hull. In 2015, Iceland and Romania were located at the points of extrema. Luxembourg was an "extreme" country on both coordinates. In 2020, Ireland, Malta, Bulgaria and Luxembourg were at the points of extrema.

In 2005, there were five countries on the left side of the primary hull ($\lambda_1^{min} = 0.2$), three countries – on the right side ($\lambda_1^{max} = 0.33$), four countries each – on lower and upper sides ($\lambda_2^{min} = \lambda_2^{max} = 0.25$). In 2010, the left side of the hull was formed by four countries ($\lambda_1^{min} = 0.25$), the right side – by three countries ($\lambda_1^{max} = 0.33$). It was the same on lower and upper sides ($\lambda_2^{min} = 0.25$, $\lambda_2^{max} = 0.33$). In 2015, the picture changed, and there were three countries on the left side ($\lambda_1^{min} = 0.33$), and four countries on the right side ($\lambda_1^{max} = 0.25$). The lower side was formed by five countries ($\lambda_2^{min} = 0.2$), and the upper side – by two countries ($\lambda_2^{max} = 0.5$). In 2020, there were four countries each on left and lower sides ($\lambda_1^{min} = \lambda_2^{max} = 0.25$), and three countries each on right and upper sides ($\lambda_1^{min} = \lambda_2^{max} = 0.33$).

Next, the vertices of two secondary hulls were determined based on the coordinates of the vertices of the primary hull and λ weighting factors. Distance indices according to the first coordinate (ι_1) were calculated as the ratio of the segments of horizontal chords connecting left and right horizons. Distance indices according to the second coordinate (ι_2) were calculated as the ratio of the segments of vertical chords connecting lower and upper horizons. The results of these calculations are presented in Table 2.

Table 2. Coordinates of countries on the plane of distance indices

	2005		2010		2015		2020	
	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
Austria	0.6005	0.4060	0.1733	0.4024	0.1532	0.5893	0.3935	0.5491
Belgium	0.3431	0.4294	0.0667	0.5402	0.0333	0.7680	0.3069	0.6039
Bulgaria	0.7631	0.8939	0.4775	0.9589	0.3580	0.9874	0.5682	1.0000
Croatia	0.8555	0.7976	0.3126	0.7875	0.4030	0.8654	0.6480	0.9179
Cyprus	0.9405	0.7382	0.2982	0.5703	0.5050	0.7625	0.8239	0.7053
Czechia	0.6272	0.7177	0.1950	0.7173	0.2276	0.8311	0.4535	0.7759
Denmark	0.5112	0.5243	0.1198	0.5145	0.0416	0.7635	0.1850	0.7419
Estonia	0.9977	0.3005	0.2546	0.4987	0.2330	0.7500	0.4804	0.7067
Finland	0.5815	0.6648	0.0944	0.6594	0.1092	0.7911	0.4194	0.7121
France	0.5647	0.4090	0.1832	0.4234	0.1967	0.6218	0.5371	0.5321
Germany	0.4811	0.4491	0.1153	0.4712	0.0603	0.6821	0.2277	0.6082
Greece	0.8974	0.4684	0.2878	0.5034	0.4880	0.6943	0.9709	0.7568
Hungary	1.0000	0.6336	0.2786	0.6535	0.3365	0.7780	0.5814	0.7171
Iceland	0.9091	0.6432	1.0000	0.6769	1.0000	0.8922	0.3270	0.6886
Ireland	0.7432	0.7421	0.2722	0.6339	0.5961	0.7850	1.0000	0.7640
Italy	0.5997	0.5555	0.1674	0.5034	0.1859	0.6901	0.4246	0.6527
Latvia	0.7713	0.4260	0.2354	0.5863	0.3395	0.8472	0.5505	0.8176
Lithuania	0.7585	0.7855	0.3290	0.9046	0.2467	0.9869	0.4829	0.9696
Luxembourg	0.5976	0.0000	0.2124	0.0000	0.0000	0.0000	0.2100	0.0000
Malta	0.5150	1.0000	0.1820	1.0000	0.0372	0.9819	0.0000	0.7678
Netherlands	0.5407	0.4711	0.1381	0.4885	0.0228	0.8775	0.0705	0.6858
Poland	0.7465	0.7586	0.2575	0.8154	0.3934	0.8987	0.5801	0.8547
Portugal	0.9695	0.9171	0.4541	0.9202	0.6319	0.9754	0.8999	0.9482
Romania	0.6715	0.8463	0.4441	0.9921	0.2416	1.0000	0.6418	0.9564
Slovakia	0.7199	0.6742	0.2116	0.6813	0.3074	0.8026	0.6544	0.7709
Slovenia	0.5530	0.4912	0.2119	0.4708	0.2290	0.6796	0.4794	0.6155
Spain	0.8519	0.3361	0.3553	0.3292	0.4141	0.5688	0.7405	0.4740
Sweden	0.6937	0.3383	0.2137	0.3515	0.1688	0.5522	0.4228	0.4847
Switzerland	0.0000	0.1567	0.0000	0.1154	0.0125	0.7679	0.0363	0.2650
Un. Kingdom	0.5320	0.5439	0.1506	0.5294	0.1701	0.6765	0.4372	0.6201

Notes: i_1 – an index of the distance to the horizon of the maximum of the net international investment position; i_1 – an index of the distance to the horizon of the maximum of the net international immigrant stock. The states forming the hull on the plane of indices are highlighted in gray

Source: prepared by the authors based on data from statistical sites

At the next stage of the research, international states were built. The central C states were initially constructed on the z_1Oz_2 plane by the method of successive subtraction of hulls from the studied set of countries. According to this method, the original primary Conv$\mathbb{X}$ hull was considered "zero" one and subsequent hulls were considered "negative" ones. In 2005, there were four such "negative" hulls with –4 being the Italy – Slovenia segment. Thus, the central state of 2005 represented the middle of this segment. In 2010, there was the same number of "negative" hulls, but the Slovenia – Italy – United Kingdom triangle was the last one. In 2015 and 2020, Estonia was the central state.

By the same method, but already on the i_1Oi_2 plane, focused F states were built. In 2005-2015, there were four "negative" hulls on the plane of indices. In 2005, the Slovenia – Ireland – Italy triangle was the last hull, in 2010 – the Latvia – Ireland segment was. In 2015, the Slovenia – Italy – Estonia – Latvia – Slovakia pentagon was the last. In 2020, Estonia turned out to be the focused state.

General G states were constructed on the plane of initial indicators as points with $(\overline{z}_1; \overline{z}_2)$ coordinates, and then mapped onto the plane of indices. Relative R states were constructed in the opposite order. First, the $(\overline{i}_1; \overline{i}_2)$ point was determined on the plane of indices, which was then mapped onto the plane of initial indicators. The zero Z state on the plane of initial indicators was the origin of the coordinates, but on the plane of indices its coordinates were turned into variables. In turn, the symmetrical S state on the plane of indices was the point of intersection of unit square diagonals, and on the plane of initial indicators its coordinates became variable. The coordinates of international states are given in Table 3.

Table 3. Coordinates of international states on the planes of initial indicators and indices

International state	2005		2010		2015		2020	
Central state	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
	−12.1961	3.8941	−24.8275	6.0195	−42.5924	0.4960	−21.7703	−0.6507
	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
	0.5761	0.5224	0.1761	0.5015	0.2330	0.7500	0.4804	0.7067
Focused state*	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
	0.6319	0.5963	0.2538	0.6101	0.2590	0.7539	0.4804	0.7067
General state	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
	−24.1654	1.9141	−59.1838	2.4680	−44.6703	3.3138	−15.1918	4.3274
	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
	0.371	0.5595	0.2042	0.5757	0.2477	0.7040	0.4634	0.6492
Relative state	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
	−31.6870	1.2371	−98.2570	1.8415	−54.1820	−0.8020	−22.8154	0.4832
	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
	0.6779	0.5706	0.2564	0.5900	0.2714	0.7622	0.4851	0.6887
Symmetric state	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
	2.5127	5.5590	−238.0793	5.0358	−108.794	15.3101	−12.1159	15.3894
	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Zero state	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	i_1	i_2	i_1	i_2	i_1	i_2	i_1	i_2
	0.4917	0.6205	0.1119	0.6320	0.1271	0.7953	0.4043	0.7381

Notes: z_1 – the net international investment position at the end of the second quarter as a percentage of the country's GDP; z_2 – the net international immigrant stock as a percentage of the country's population excluding migrants; i_1 – the index of the distance to the horizon of the maximum of the net international investment position; i_2 – the index of the distance to the horizon of the maximum of the net international immigrant stock. The coordinates of the focused state on the z_1Oz_2 plane are not calculated

Source: prepared by the authors based on data from statistical sites

The distance of an individual country to its nearest neighbors and certain international states is an important characteristic of its international position. Thus, during two periods, Austria – France (2005, 2010), Austria – Sweden (2015, 2020), Italy – United Kingdom (2015, 2020), Hungary – Slovakia (2015, 2020), Luxembourg – Switzerland (2010, 2020) were mutually nearest neighbors. In some years, triangles of mutually nearest neighbors: Ireland – Poland – Lithuania (2005), Bulgaria – Portugal – Romania (2010), Croatia – Poland – Lithuania (2010), Belgium – Denmark – Switzerland (2015), Cyprus – Greece – Ireland (2015) were formed. Some countries were not the nearest neighbors for any other one. In 2005, there were eight such countries. Of these, Italy, Slovakia, and Greece were in the inner region bounded by the hull, and the rest were on the hull itself. In 2010, there were six such countries, three of which (Cyprus, Slovenia, Spain) were located in the internal region. In 2015, nine countries were not the nearest countries to the remaining ones. At the same time, only two such countries (Iceland and Luxembourg) were on the hull. In 2020, eight countries were not the nearest neighbors of the others. Of these, three countries (Lithuania, Portugal and Spain) were located on the hull.

Nearest neighbors were also determined for international states. In 2005, Denmark (d = 0.0268) was the nearest to the symmetrical state, Slovenia (d = 0.0388) – to the central state, the United Kingdom (d = 0.0865) – to the zero state. Italy was the nearest neighbor of three states at once – relative (d = 0.0797), general (d = 0.0376) and focused (d = 0.0520) ones. In 2010, Latvia was the nearest neighbor of relative and general states (d = 0.0213, d = 0.0330). Greece (d = 0.2123) was the nearest neighbor of the symmetrical state, Italy (d = 0.0088) – of the central state, Finland (d = 0.0325) – of the zero state, and Ireland (d = 0.0301) – of the focused state. In 2015, Estonia was the nearest neighbor of relative and focused states (d = 0.0404, d = 0.0263), and was itself a central international state (d = 0). As in the previous period, Finland was the nearest country to the zero state (d = 0.0183). Slovenia was the nearest country to the general state (d = 0.0307), and Spain – to the symmetrical state (d = 0.1101). In 2020, Estonia was again the nearest neighbor of the relative state (d = 0.0186), and itself was at the same time the central and focused international state (d = 0). Finland was again the nearest country for the zero state (d = 0.0301) and Slovenia – for the general state (d = 0.0373). In this year, France (d = 0.0490) was the nearest neighbor of the symmetrical state. Mean distances of countries to various international states in one year characterise the geometry of the studied set (Table 4).

Table 4. Mean distances on the plane of indices

	Mean distance to the international state						Distance to other countries		
	to central one	to focused one	to general one	to relative one	to symmetrical one	to zero one	mean to national averages	mean to minimum ones	mean to maximum ones
2005	0.2826	0.2725	0.2716	0.2709	0.3151	0.3256	0.3866	0.1040	0.8870
2010	0.2422	0.2370	0.2341	0.2356	0.3757	0.2713	0.3491	0.0857	0.8739
2015	0.2342	0.2347	0.2419	0.2352	0.4307	0.2603	0.3463	0.0877	0.9497
2020	0.2707	0.2707	0.2746	0.2714	0.3378	0.2852	0.4031	0.0978	0.8737

Source: prepared by the authors based on data from statistical sites

The mean distance of countries to a certain international state in different years is an important characteristic of the dynamics of the studied set. These distances are shown on the left side of Table 4. As these calculations show, in each year mean distances of countries to the central C state, the focused F state, the general G state and the relative R state are about the same. On the other hand, these distances change little over time. Given this, it is logical to construct the main M state, the coordinates of which are mean coordinates of these four states:

$$\begin{aligned} i_1(M) &\stackrel{\text{def}}{=} (i_{1C} + i_{1F} + i_{1G} + i_{1R})/4 \\ i_2(M) &\stackrel{\text{def}}{=} (i_{2C} + i_{2F} + i_{2G} + i_{2R})/4 \end{aligned}. \tag{15}$$

Mean distances of countries to the main state will be equal to: $\overline{d_{2005}(X,M)} = 0.2719$, $\overline{d_{2010}(X,M)} = 0.2328$, $\overline{d_{2015}(X,M)} = 0.2354$, $\overline{d_{2020}(X,M)} = 0.2711$. These distances, as well as the distances to it of zero and symmetric ($d(Z, M)$ and $d(S, M)$) states can be used in a comparative analysis of different sets of countries (or different indices of the same set).

The right part of Table 4 shows mean mutual distances of the countries. As the obtained results show, the arithmetic mean of mean national distances to other countries is quite high all years and is equal to about a quarter (or more) of the maximum possible distance $d^{max} = \sqrt{2}$. Mean distance of the countries to their farthest neighbor is more than twice as long, and mean distance to their nearest neighbor is about four times smaller.

There should be a positive relationship between the countries' distances to relative $R(\overline{i}_1; \overline{i}_2)$ and general $G(\overline{z}_1; \overline{z}_2)$ states. This follows from the methods of their construction. Thus, in order to construct the G state on the plane of indices, z_1, z_2 initial indicators of the countries are first averaged, and then their min-max normalisation is carried out. To construct the R state, the min-max normalisation of the indicators of each individual country is first carried out, and then they are averaged. If the normalisation of initial indicators is carried out only in relation to actual extrema of the studied set, then both states would coincide. Normalisation of secondary hulls according to the proposed method leads to their divergence. On the plane of $d(X_k, R)Od(X_k, G)$ distances, observation points are located on both sides of the bisector.

The parameters of linear regression equations based on the data of one year are shown in the upper left part of Table 5. As the calculations show, the a_1 parameter is close to one, and the a_0 parameter is close to zero. At the same time, $a_1 < 1$ corresponds to a positive value of the a_0 parameter, and $a_1 > 1$ corresponds to a negative value of this parameter. As a result, the sum of parameters is close to one. Given this, the $|1 - (a_0 + a_1)|$ difference can be used as an indicator of the influence of normalisation by the method of secondary hulls. The lower left part of Table 5 shows regression parameters in which the distances are replaced by their increments. In these equations, the a_0 parameter is also close to zero, and a_1 is close to one. There is no unambiguous correspondence between the sign of the a_0 parameter and the a_1 deviation from one, but the sum of these parameters is also close to one.

Table 5. Equation of linear correlation between countries' distances to international states on the plane of indices

Distances according to data of one year	Independent variable – the country's distance to the relative state (state of mean indices)							
	Dependent variable – the country's distance to the general state (state of mean indicators)				Dependent variable – the mean country's distance to other countries			
	a_0	a_1	$\sum a$	R^2	a_0	a_1	$\sum a$	R^2
d_{2005}	−0.0020	1.0100	1.0080	0.9585	+0.1758	0.7781	0.9539	0.9704
d_{2010}	−0.0108	1.0397	1.0289	0.9657	+0.1702	0.7593	0.9295	0.9608
d_{2015}	+0.0121	0.9768	0.9889	0.9287	+0.1567	0.8060	0.9627	0.9657
d_{2020}	+0.0074	0.9848	0.9922	0.9611	+0.2142	0.6963	0.9105	0.9639
Increments of the first, second and third orders	Independent variable – the increase in the country's distance to the relative state (state of mean indices)							
	Dependent variable – the increase in the country's distance to the general state (state of mean indicators)				Dependent variable – the increase in the mean country's distance to other countries			
	a_0	a_1	$\sum a$	R^2	a_0	a_1	$\sum a$	R^2
$\Delta^{(1)}_{05/10}$	−0.0013	1.0242	1.0229	0.9945	−0.0104	0.7686	0.7582	0.9209
$\Delta^{(1)}_{10/15}$	+0.0081	0.9390	0.9471	0.9289	−0.0025	0.7231	0.7206	0.9234
$\Delta^{(1)}_{15/20}$	−0.0037	1.0056	1.0019	0.9810	+0.0286	0.7814	0.8100	0.9563

Continued Table 5.

Increments of the first, second and third orders	Independent variable – the increase in the country's distance to the relative state (state of mean indices)							
	Dependent variable – the increase in the country's distance to the general state (state of mean indicators)				Dependent variable – the increase in the mean country's distance to other countries			
	a_0	a_1	$\sum a$	R^2	a_0	a_1	$\sum a$	R^2
$\Delta^{(1)}_{05/15}$	+0.0099	1.0104	1.0203	0.9717	+0.0101	0.7038	0.7139	0.9001
$\Delta^{(1)}_{10/20}$	−0.0093	0.9384	0.9291	0.9441	+0.0303	0.8040	0.8343	0.9560
$\Delta^{(1)}_{05/20}$	−0.0217	0.9378	0.9161	0.9297	+0.0238	0.7084	0.7322	0.9123

Source: prepared by the authors based on data from statistical sites

There should also be a positive relationship between the country's distance $d(X_k, R)$ to the relative state and its mean distance $\overline{d(X_k, X_{j\neq k})}$ to other countries. This can be illustrated by the example of a regular polygon. In it, mean distances of each vertex to the others are equal. The point of mean coordinates of the vertices coincides with the centre of symmetry of the polygon, and the distances of the vertices to it are smaller than their mean distances from each other. The mean distance can be represented as a linear function of the distance to the centre of symmetry: $\overline{d(X_k, X_{j\neq k})} = a_1 d(X_k, R) + a_0$. In the case of a unit square, this function will have the form: $\overline{d(X_k, X_{j\neq k})} = 2d(X_k, R)/3 + 2/3$. In the case of a single hexagon, this function will have the form: $\overline{d(X_k, X_{j\neq k})} = 2\sqrt{3}d(X_k, R)/5 + 4/5$.

The hull of the countries' states on the plane of indices is an irregular polygon, and the countries themselves are asymmetrically located around the state of the $R(\overline{\imath}_1;\ \overline{\imath}_2)$ mean indices. On the $(X_k, R)O\overline{d(X_k, X_{j\neq k})}$ plane of distances, all observation points are located above the bisector. As the calculations show (the upper right part of Table 5), in regression equations based on the data of one year, the a_1 parameter ranges from 0.7 to 0.81, and the a_0 parameter ranges from 0.16 to 0.21. There is no unequivocal correspondence between these parameters, but their sum changes little – from 0.91 to 0.96. On this basis, the sum of these parameters can be used as an indicator of the deviation of the observed states from certain idealised polygons (not necessarily regular ones). As a model, it is possible to use, for example, the following $\mathfrak{B}$ set of reference states, the hull of which coincides with the hull of the observed states, and the successive subtraction of its hulls forms similar figures:

$$\mathrm{Conv}\mathfrak{B} \overset{\mathrm{def}}{=} \mathrm{Conv}\mathbb{X}. \tag{16}$$

$$\mathrm{Conv}(\mathfrak{B} \setminus \mathrm{Conv}\mathfrak{B}) \sim \mathrm{Conv}\mathfrak{B}, \ldots \tag{17}$$

In correlation equations between distance increments, the range of values of the a_1 parameter almost does not change (lower right part of Table 5). Instead, the values of the a_1 parameter become an order of magnitude smaller, and some of them become negative.

Table 6 shows the distances between main pairs of international states of the same name. As it is shown, there is a correlation between the countries' distances to relative $R(\overline{\imath}_1;\ \overline{\imath}_2)$ and general $G(\overline{z}_1;\ \overline{z}_2)$ states. Accordingly, the $d(G, R)$ distance between these states is small (in the range of 0.04-0.06). This distance can be used as an additional indicator of the impact of normalisation using the secondary hulls method.

Table 6. Distances on the plane of indices between different international states of the same year

	$d(C, F)$	$d(C, Z)$	$d(F, S)$	$d(G, R)$	$d(G, Z)$	$d(R, S)$
2005	0.0926	0.1294	0.1633	0.0423	0.1577	0.1914
2010	0.1336	0.1455	0.2697	0.0541	0.1081	0.2597
2015	0.0263	0.1152	0.3501	0.0629	0.1513	0.3479
2020	0.0000	0.0824	0.2076	0.0452	0.1068	0.1893

Notes: C – central state; F – focused state; G – general state; R – relative state; S – symmetrical state; Z – zero state

Source: prepared by the authors based on data from statistical sites

Another pair of interconnected states is formed by central and focused states – $C(\overline{z}_1(\mathbb{C});\ \overline{z}_2(\mathbb{C}))$, $F(\overline{\imath}_1(\mathbb{F});\ \overline{\imath}_2(\mathbb{F}))$. These states are built according to the same method of successive subtraction of hulls, so the distance between them is the result of the transition from the z_1Oz_2 plane of original indicators to the i_1Oi_2 plane of indices. For these reasons, the $d(C, F)$ distance can be used as an indicator of the transformation of the original coordinate system. In 2010, after the global crisis, the distance between central and focused states reached a maximum. In 2015, it began to decrease and in 2020 it reached the theoretical minimum – $d_{2020}(C, F) = 0$. Estonia became the country for which these states coincided.

Remaining pairs of the states presented in Table 6 are chosen according to the plane on which they have been originally constructed. Namely, on the z_1Oz_2 plane, the zero $Z(0;\ 0)$ state is the main stable point, and on the i_1Oi_2 plane, the symmetric $S(0.5;\ 0.5)$ state is such a point. General and central states are initially built on the z_1Oz_2 plane, therefore Table 6 presents their distances to the zero state – $d(G, Z)$ and $d(C, Z)$. Relative and focused states are initially built on the i_1Oi_2 plane, therefore Table 6 presents their distances to the symmetric state – $d(R, S)$ and $d(F, S)$. During 2005-2020, the distances $d(G, Z)$ and $d(C, Z)$ changed little in absolute terms. This can be explained by the fact that on the z_1Oz_2 plane, the trajectories of general and central states are "shorter" than the trajectories of other international states. Instead, the $d(R, S)$ and $d(F, S)$ distances varied quite strongly, but in the same direction and by approximately the same amount. As a result, in 2015, they almost equaled each other ($d \approx 0.35$). This is explained by the fact that this year the coordinates of relative and focused states on the plane of indices almost coincided.

Table 7 shows distances of an alternative type – between international states of the same name in different years. As the calculations show, the global crisis of 2008 led to a significant shift of all international states – central C, focused F, general G, relative R and zero Z ones. In 2015, the shifts of these states continued, but already by a smaller amount. Shifts continued in 2020, and for all states except the central one, they turned out to be larger than in the previous period. In general, according to the results of 2005-2020, the zero Z state ($d \approx 0.15$) shifted the least, and the relative R state ($d \approx 0.23$) – the most. This can be explained by the fact that on the plane of indices, the zero state is a function of the states of the countries forming the hull, and the relative state is a function of the states of all countries.

Table 7. Distances on the plane of indices between international states of the same name in different years

	2005-2010	2005-2015	2005-2020	2010-2015	2010-2020	2015-2020
Central state	0.4006	0.4118	0.2077	0.2549	0.3671	0.2512
Focused state	0.3784	0.4049	0.1875	0.1439	0.2463	0.2264
General state	0.4332	0.4153	0.1955	0.1355	0.2694	0.2225
Relative state	0.4219	0.4494	0.2261	0.1729	0.2491	0.2260
Zero state	0.3799	0.4043	0.1465	0.1640	0.3110	0.2830

Source: prepared by the authors based on data from statistical sites

At the last stage of the study, the areas of the figures in a unit square are calculated. The middle part of Table 8 presents the areas of the figures bounded by adjacent sides of the unit square and the $\mathrm{Conv}\mathbb{X}(i_1; i_2)$ hull of observed states The large relative size of the area of the upper left outer figure (> 33%) was the main feature of 2005. As a result, the sum of upper outer areas was more than twice the sum of lower outer areas, and the sum of left outer areas was three times larger than the sum of right ones. The first relation shows that in 2005 the $\mathrm{Conv}\mathbb{X}(i_1; i_2)$ hull was much closer to the horizon of the maximum of the net migrant stock than to the horizon of the corresponding minimum. The second relation shows that $\mathrm{Conv}\mathbb{X}(i_1; i_2)$ hull was much closer to the horizon of the minimum of the net investment position than to the horizon of its maximum.

Table 8. The area of the figures formed by the hulls of countries' states on the plane of indices

Year	Area of the region limited by the hull	Area of the figure outside the hull				Areas of intersection of regions limited by the hull			
		Upper left	Upper right	Lower right	Lower left	2005 & 2010	2010 & 2015	2015 & 2020	All years
2005	0.6491	0.2172	0.0257	0.0612	0.0468	0.3938	–	–	0.2451
	100%	33.46%	3.96%	9.43%	7.21%	60.66%	–	–	37.75%
2010	0.5573	0.0709	0.0930	0.2665	0.0123	0.3938	0.4008	–	0.2451
	100%	12.71%	16.69%	47.82%	2.20%	70.65%	71.91%	–	43.97%
2015	0.5124	0.0124	0.0292	0.4461	0.00	–	0.4008	0.4305	0.2451
	100%	2.42%	5.69%	87.07%	0.00%	–	78.22%	84.03%	47.83%
2020	0.5841	0.0647	0.0230	0.2864	0.0418	–	–	0.4305	0.2451
	100%	11.08%	3.94%	49.02%	7.15%	–	–	73.70%	41.95%

Source: prepared by the authors based on data from statistical sites

The global financial crisis of 2008 led to a radical shift in the hull. According to the data of 2010, the lower right outer area became the largest one (> 47%). As a result, the sum of lower outer areas became 1.7 times larger than the sum of upper ones, and the sum of right areas became almost 3.9 times larger than the sum of left ones. Thus, the $\mathrm{Conv}\mathbb{X}(i_1; i_2)$ hull significantly approached the horizon of the minimum of the net migrant stock and the horizon of the maximum of the net investment position. This trend further strengthened in 2015. In 2020, the lower right outer area practically returned to its post-crisis state (> 49%). Four right columns of Table 8 show the values of cross-sectional areas of the regions bounded by the hulls of different years. According to these data, the biggest changes took place in 2010. The area of intersection of the regions in 2005 and 2010 was about 60% of the area of 2005 and about 70% of the area of 2010. In other words, in 2010, 40% of the area of 2005 was lost and 30% of it was renewed. The rightmost column of Table 8 shows that the "stability zone" (the intersection of the regions bounded by the hulls of all years) changed little, from ≈ 38% relative to 2005 to ≈ 42% relative to 2020.

Discussion

As it has been shown, the asymmetry of the international position of the countries can be determined with the help of elementary min-max normalisation of initial indicators and the construction of a convex closure of the data. In modern literature, coefficients of skewness, in particular, which characterise a statistical distribution rather than a separate studied unit, are used as a characteristic of asymmetry. Pearson's and Bowly's coefficients are classical measures of skewness. At the same time, new measures are being developed. Thus, M.A. Eltehiwy & A.-B.A. Abdul-Motaal (2020) have proposed a new coefficient of skewness for grouped data. The coefficient proposed by them is built on the basis of the summation of cumulative frequencies of data classes. As the authors note, the advantage of the new coefficient is that it is bounded by ± 1. The effectiveness of this coefficient is evaluated by comparing it with classical measures of skewness. Mean square error (MSE) and mean absolute error (MAE) are used for simulation.

As for normalisation, in modern studies it serves as a tool for more complex comparisons, rather than their end point. Usually, transformed data are used to construct various composite indices and in multicriteria analysis. Thus, E. Mazur-Wierzbicka (2021) has analysed the transition of the countries of the European Union to a circular economy. At the first stage of this study, initial data for each attribute are rescaled using the min-max normalisation method. Next, the normalised data are standardised and, based on their matrix, a pattern object – a virtual model country – is determined. For the stimulating attribute, its maximum value is chosen as the coordinate of this country, and in the case of the destimulating attribute, its minimum value is chosen. Then the Euclidean distances of real countries to the model country are calculated. For obtained distances, the arithmetic mean and the standard deviation are determined.

R. Trishch *et al*. (2023), in turn, have proposed a fundamentally new approach to ranking countries according to the level of their economic development. They have tested their approach on the data of the European Union countries. As the authors have established, the non-linear division of countries into homogeneous groups is a more accurate mapping of the current situation and can be used in various ratings. S. Jednak *et al*. (2018) have compared the results of the ranking of the countries of Southeast Europe using three different methods – classification by income per capita (according to the World Bank methodology), classification by the Human Development Index and I-distance (multivariate statistical analysis method, developed by Ivanovic). Unlike these authors, the method of secondary hulls presented in the proposed study has been used primarily to diagnose the state of countries in a certain period of time and changes in the state of the studied set.

The problem faced by the authors of the presented study is that due to the relatively low frequency of publication of reports on population migration, the trajectories of countries' movements become too discrete. However, in the future, the gradual accumulation of data will create more favorable conditions for studying the geometry of such trajectories. In this regard, the article by A. Milaghardan *et al*. (2018) is of considerable interest. As its authors have shown, using the traditional method of the convex hull of data, it is possible to detect important geometric properties of 2D trajectories – self-intersecting, turning and curvature points. They have demonstrated their method using the example of a set of points registered by GPS (Global Positioning System).

Non-parametric methods are a certain alternative to stochastic analysis. The use of a convex hull of data is the most well-known method of non-parametric analysis of various subjects (or objects). Geometrically, the state of the studied elements is mapped as a point on the plane of certain indicators (or in their multidimensional space). Data Envelopment Analysis (DEA) is a variant of the convex hull approach. DEA is a set of linear programming problems in which the relative distance of points to the efficient part of the convex hull is calculated. Initially, it has been applied in operations research to determine the degree of efficiency of decision-making units (DMUs). Over time, DEA has branched out more and more and has been supplemented with new models. Thus, O. Despić (2013) has provided a brief overview of the advantages of the geometric model of efficiency and showed where it fits in relation to classical DEA models. A detailed overview of the history and current trends in the use of DEA has been presented in the works of A. Panwar *et al*. (2022).

The complication of the DEA mathematical apparatus is one of the trends. Frameworks with fuzzy variables (Al-Refaie & Lepkova, 2023), theories of neutrosophic and hypersoft sets (Jafar *et al*., 2022) are used. A synthesis of convex hull methods and microeconomic analysis is also taking place (Radovanović *et al*., 2022; Hyder *et al*., 2023). It should be noted that the results of DEA depend significantly on the method of units ranking. Further studies have revealed certain shortcomings of classical DEA. To overcome them, various alternative methods have been proposed (Dehnokhalaji *et al*., 2017; Tavana *et al*. 2021; Varelas *et al*., 2022). Based on the results of Monte Carlo simulations, M. Zarrin & J.O. Brunner (2023) have concluded that AR (Assurance Region) and SBM (Slacks-Based Measurement) are the best models. M. Farahmand & M.I. Desa (2017) have reviewed DMU ranking methods using DEA. To solve the problem of choosing weights, the authors propose a model that doesn't not depend on DEA and linear programming methods. The article by R. Rani *et al*. (2018) proposes a combination of the DEA cross-efficiency method and the maximum-minimum principle to determine the optimal operator allocation in one company.

From the end of the 20th century, DEA methods begin to be used at the macro level – to compare the national economies of different countries. A systematic review of relevant literature on national innovation systems is presented in the work of E. Narayanan *et al*. (2022). The world (or global) technology frontier – the boundary of the efficiency of the use of production factors for the studied group of countries – is the most famous example of DEA macro models. A concise review of relevant literature is presented in the article by I. Zagoruiko & L. Petkova (2022). In general, in modern studies, convex hulls are used to compare countries on many different indicators. However, in most such studies, only the effective part of the convex hull is used. Thus, G. Anderson *et al*. (2008) have applied the lower convex hull approach in the study of the problem of poverty. V. Holý (2024) proposes to compare the state of higher education of the studied countries using the dynamic ranking method. This method has a stochastic nature and, as the author notes, complements the usual models of the second stage of DEA, in which efficiency factors are searched for and measured.

S. Athanassoglou (2016) has proposed an alternative approach to construct a sustainable energy index for the worst-case DEA. According to this approach, the new model is to maintain the original objective function and its constraints. As in classical DEA, an agent score is given by the corresponding weighted sum of non-normalised indicators. The ratio of the agents' scores to the score of the best of them is the yardstick of performance. However, the problem of maximisation of the performance function turns into the problem of its minimisation. A linear programming problem that determines the "most favorable" weights for a certain agent is called the "benefit of the doubt" method for composite indicators. Using this method, E. Lafuente *et al*. (2022) have conducted a comparative analysis of the entrepreneurial ecosystems of 71 countries for the period from 2016. Against this background, the work of M. Funke & M. Gronwald (2009), in which the entire convex hull of the studied countries has been used, also stands out. The article examines the impact of trade openness on economic growth. To characterise the difference between data points, the authors have used Gower's distance. Their analysis has found that some African countries do not pass the convex hull test. The difference between the proposed study and the study conducted by M. Funke & M. Gronwald (2009) is that convex hulls have not been used to check the reliability of statistical data (they are exemplary in the countries of the European Union), but for 2D diagnostics of the states of the countries under study.

In the context of the proposed study, the article by S. Rakhshan (2017) is of particular interest. The author has proposed a new method of units ranking, which he calls TOPSIS-DEA. According to this method, two boundaries are constructed – efficient frontier and anti-efficient frontier. As the author demonstrates, both frontiers can intersect at the points of two diagonally opposite DMUs, characterised by the maximum of one indicator and the minimum of the other one. On the "$x_1 / y - x_2 / y$" plane of specific inputs, the efficiency value of the DMU located between two frontiers is the ratio of the length of the radius of the projection point on the efficient frontier to the length of the radius of the DMU itself. The author interprets the inefficiency coefficient in a similar way. The method of secondary hulls proposed in the study can also be applied to the geometric model of S. Rakhshan (2017). In Figure 2, this method is applied to the world technology frontier (WTF), which intersects with the world technology anti-efficient frontier.

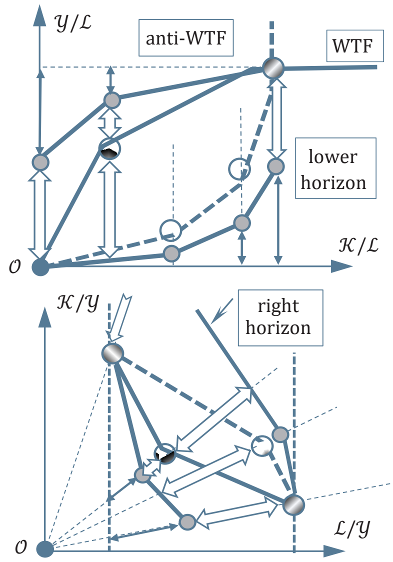

Figure 2. A case of crossing the world technology frontier and the world technology anti-efficient frontier

Notes: $\mathcal{Y}$ – the real volume of national production; $\mathcal{K}$, $\mathcal{L}$ – values of capital and labour; WTF – world technology frontier; anti-WTF – world technology anti-efficient frontier

Source: authors' model based on S. Rakhshan (2017), E. Lafuente *et al*. (2020), I. Zagoruiko & L. Petkova (2022)

On the left half of Figure 2, the frontiers are shown in the coordinate system "($\mathcal{K}/\mathcal{L}$) capital-labour ratio – ($\mathcal{Y}/\mathcal{L}$) labour productivity". To construct the efficient frontier, the set of countries is supplemented by a $O(0; 0)$ "country of origin" and a $\mathcal{I}(\infty; \max(\mathcal{Y}/\mathcal{L}))$ "country at infinity". As a result, WTF will represent the upper part of the hull of the augmented set of countries.

On the "$\mathcal{K}/\mathcal{L} - \mathcal{Y}/\mathcal{L}$" plane, the upper horizon is a polygonal chain of weighted arithmetic means of the global (actual) maximum of labour productivity and the theoretical maximum at the world technology frontier. The lower horizon is a line of similar means of the zero level of labour productivity and the level of productivity at the world technology anti-efficient frontier. The country's distances to these opposite horizons are measured along vertical chords.

On the right half of Figure 2, the efficient and anti-efficient frontiers are plotted in the "($\mathcal{L}/\mathcal{Y}$) labour-to-GDP ratio – ($\mathcal{K}/\mathcal{Y}$) capital-to-GDP ratio" coordinate system. To construct the efficient frontier, the set of countries is supplemented by two countries at infinity – $\mathcal{I}_{\mathcal{K}}(\infty; \min(\mathcal{K}/\mathcal{Y}))$ and $\mathcal{I}_{\mathcal{L}}(\min(\mathcal{L}/\mathcal{Y}); \infty)$. So, WTF will represent the lower left part of the hull. According to the geometric form, this frontier is an analogue of the isoquant of the production function.

On the "$\mathcal{L}/\mathcal{Y} - \mathcal{K}/\mathcal{Y}$" plane, the left horizon is a line of means of the global (actual) minimum of labour intensity and the theoretical minimum at the world technology frontier. The right horizon is a line of means of the global maximum of labour intensity and the theoretical maximum at the world anti-efficient frontier. The country's distances to these opposite horizons are measured along radial chords.

The proposed method of secondary hulls allows calculating two asymmetry indices of the country's position, regardless of whether efficient and anti-efficient frontiers intersect and how the distance between them is measured. At the same time, each country will be characterised by a unique pair of asymmetry indices. This is the important difference between the proposed method and the DEA method, according to which all countries on the boundary of efficiency are characterised by a unit distance.

The geometric character of the method of secondary hulls creates additional opportunities for its application. Thus, in addition to comparing the real state of a certain country with other countries, the analysis of the effects of a virtual change in one of its indices or one of its coordinates is an important direction of the proposed research. Algebraically, the set of interconnected virtual states of two A and B countries can be represented in the form of a matrix:

$$\mathcal{V}(A,B) = \begin{pmatrix} f(z_{1A}, \imath_{2B}) & f(z_{1B}, \imath_{2A}) \\ f(z_{2A}, \imath_{1B}) & f(z_{2B}, \imath_{1A}) \end{pmatrix}. \qquad (18)$$

The elements of the $\mathcal{V}(A, B)$ matrix are the points of intersection of the chord of one country with the cross-section on which the other country is located. In the first row of $\mathcal{V}(A, B)$ matrix, the states located on vertical chords are recorded, in the second row – those located on horizontal chords. In the first column, the states located on the chords of A country are recorded, in the second row – those located on the chords of B country. From a geometric point of view, each index characterises the line of chord sections: "vertical" section passing from the highest to the lowest chord corresponds to the $\imath_1$ = const condition, "horizontal" section located between extreme lateral chords corresponds to the $\imath_2$ = const condition.

With the help of matrices of virtual states, various international comparisons can be made. Thus, it is possible to use the $\mathcal{V}\left(X_k, \overline{\mathbb{X}\backslash X_k}\right)$ matrix, in which the studied X_k country is the A country, and mean indicators of the set of countries from which the studied country is excluded are the indicators of the B country. Analogous $\mathcal{V}_k(t_1, t_2)$ matrix consists of elements that are functions of coordinates and indices of one country in different time periods. The $\mathcal{V}_k(t_1, t_2) - \mathcal{A}_k(t_1, t_2)$ difference of this matrix and the matrix of actual states will characterise the effects of the change in X_k country indices, and the $\mathcal{V}_k(t_1, t_2) - \mathcal{A}_k(t_1, t_2)$ difference will characterise the effects of the change in its coordinates.

For the hull subtraction operation, there is an inverse operation of sequential construction of sets that include previous ones. This method can be used to characterise the "entourage" of a certain country (or a group of countries or international states):

$$\begin{array}{c} \mathbb{X}_{+1}(X_k)\backslash \mathrm{Conv}\mathbb{X}_{+1}(X_k) \stackrel{\mathrm{def}}{=} X_k \\ \mathbb{X}_{+2}(X_k)\backslash \mathrm{Conv}\mathbb{X}_{+2}(X_k) = \mathbb{X}_{+1} \\ \cdots \\ \mathbb{X}_{+r+1}(X_k)\backslash \mathrm{Conv}\mathbb{X}_{+r+1}(X_k) = \mathbb{X}_{+r} \end{array}. \qquad (19)$$

For a country occupying a central position in $\mathbb{X}$, $\mathbb{X}_{-s}$ sets will coincide with the corresponding $\mathbb{X}_{+r}(C)$ sets:

$$\mathbb{X} = \mathbb{X}_{+q}(C) \supset \mathbb{X}_{+q-1}(C) \supset \ldots \supset \mathbb{X}_{+1}(\mathrm{C}) \supset C. \qquad (20)$$

For the rest of the countries, these sets will differ and the last $\mathbb{X}_{+q}(X_k)$ set is a proper subset of $\mathbb{X}$:

$$\mathbb{X}_{+q}(X_k) \subset \mathbb{X}. \qquad (21)$$

For the $\mathbb{X}_{-s}$ and $\mathbb{X}_{+r}$ sets, it is also possible to construct the state of mean coordinates and the state of mean indices. Aggregations of international states or states of mutually nearest neighboring countries can serve as the centres, around which hulls of $\mathbb{X}_{+r}$ sets will be built. Like the hulls of $\mathbb{X}_{-s}$ sets, the hulls of $\mathbb{X}_{+r}$ sets can also be constructed on the $\imath_1 O \imath_2$ plane of indices. Having constructed secondary hulls for $\mathrm{Conv}\mathbb{X}(\imath_1; \imath_2)$ hull, it is possible to calculate second-order indices.

Conclusions

The main attention in the empirical part of the proposed research has been paid to the analysis of distances and areas on the $\imath_1 O \imath_2$ plane of asymmetry indices. This analysis shows the following. First, it has been confirmed that on the plane of indices there is a positive correlation between countries' distances to the $R(\bar{\imath}_1; \bar{\imath}_2)$ state of mean indices and their distances to the $G(\bar{z}_1; \bar{z}_2)$ state of mean initial indicators. Due to the fact that the sum of linear regression parameters turns out to be close to one, the $|1 - (a_0 + a_1)|$ difference can be used as an indicator of the normalisation impact by the method of secondary hulls. As such an indicator, it is also possible to use the distance between these states – $d(G, R)$. Second, the existence of a positive correlation between countries' distances to the $R(\bar{\imath}_1; \bar{\imath}_2)$ state and their mean distances to other countries has been confirmed. According to the authors, the sum of parameters of this linear regression can be used as an indicator of the deviation of the observed states from certain idealised polygons (not necessarily correct). Third, the study has shown that mean countries' distances to certain international states are approximately the same and don't change much over time. Such a regularity is found for four states – for the state of mean indices $R(\bar{\imath}_1; \bar{\imath}_2)$, the state of mean initial indicators $G(\bar{z}_1; \bar{z}_2)$ and two more states: $C(\bar{z}_1(\mathbb{C}); \bar{z}_2(\mathbb{C}))$ and $F(\bar{\imath}_1(\mathbb{F}); \bar{\imath}_2(\mathbb{F}))$, which have been constructed by the method of successive subtraction of hulls (where $\mathbb{C}$, $\mathbb{F}$ – the last non-empty sets that remain after the subtraction operation is completed). If a mean international M state is formed from these states, then countries' distances to it can be used in a comparative analysis of different sets of countries (or different indices of the same set). Similarly, it is possible to use the distances of the M state to zero and symmetric ($d(Z, M)$ and $d(S, M)$) states (where Z – the state that is the origin of the coordinates on the plane of z_1Oz_2 initial indicators, and S is the state, the indices of which are equal to 0.5). Fourth, as the study has shown, the global financial crisis of 2008 led to a radical shift in the hull of countries' states on the ($\mathrm{Conv}\mathbb{X}(\imath_1;\ \imath_2)$) plane of indices. This trend further strengthened in 2015. However, in the end, the "stability zone" (the intersection of the areas bounded by the hulls of all years) changed little.

According to the authors, the methodology presented by them can be used in further macro- and microeconomic studies. Thus, the proposed asymmetry indices can be used in various statistical studies as an additional way of normalising the initial data. The coordinates of the international state of $R(\bar{\imath}_1;\ \bar{\imath}_2)$ mean indices on the z_1Oz_2 plane of initial indicators can be used in parametric macroeconomic models, just as it is done with mean values of $\bar{z}_1$, $\bar{z}_2$ initial indicators. In models using the DEA approach, the method of secondary hulls may be the easiest way to solve the problem of negative values. At the micro level, the analogues of international states on the $\imath_1O\imath_2$ plane of indices can be used in cross-industry research, in particular, in the case when these industries are characterised by different initial indicators. The study of the dynamics of a separate macro- or microeconomic entity can be another field of application of the proposed method. In this case, its states in different periods of time will serve as observation points.

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Conflict of Interest

None.

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Індекси асиметрії міжнародного положення країн: геометричний підхід

Анотація. Статтю присвячено обґрунтуванню та апробації нового методу оцінки міжнародного положення країн. З одного боку, одним із поширених методів міжнародної компаративістики є побудова опуклої оболонки станів країн на площині певних показників. Найбільш відомим прикладом такого підходу є Data Envelopment Analysis. Зокрема, цим методом будується світовий технологічний рубіж. З другого боку, одним із універсальних методів перетворення вихідних показників є їхня нормалізація. Пропонований у статті метод поєднує побудову опуклої оболонки на площині вихідних показників з їх мін-макс нормалізацією. Метою дослідження було вимірювання відносних відстаней країн до протилежних сторін певної оболонки даних. Проблема полягає в тому, що в точках екстремумів абсолютні відстані до протилежних сторін вихідної оболонки дорівнюють нулю, а отже, відносні відстані визначити не можна. Цю проблему автори розв'язують шляхом побудови двох вторинних оболонок даних, кожна з яких дозволяє визначити індекс асиметрії за певною координатою. Протилежні сторони вторинної оболонки є середніми лініями між рівнями протилежних екстремумів та відповідними сторонами первинної оболонки. Як ваговий коефіцієнт екстремуму використовується величина, що обернена кількості країн на тій стороні первинної оболонки, на якій розташований цей екстремум. Відповідно до пропонованого методу кожна країна характеризується унікальною парою індексів асиметрії. Це відрізняє його від методу Data Envelopment Analysis, за яким усі країни на границі ефективності характеризуються одиничною відстанню. Апробацію пропонованого методу було проведено на даних щодо країн Європейського Союзу, Ісландії та Швейцарії за 2005, 2010, 2015 та 2020 роки. Як вихідні показники було обрано чисту міжнародну інвестиційну позицію (у відсотках до валового внутрішнього продукту) та різницю контингентів іммігрантів та емігрантів (у відсотках до населення країни без урахування мігрантів). Під час апробації було підтверджено існування додатного кореляційного зв'язку між певними відстанями країн на площині індексів. З'ясовано, що глобальна фінансова криза 2008 року призвела до радикального зрушення оболонки станів країн на цій площині. Відображення міжнародного стану середніх індексів на площину вихідних показників можна використовувати в економетричних моделях

Ключові слова: міжнародна компаративістика; мін-макс нормалізація; агрегатні індекси; оболонка даних; світовий технологічний рубіж; міжнародна інвестиційна позиція; міжнародна міграція